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Dimensional reduction over coset spaces and supersymmetry breaking

Pantelis Manousselis* **and George Zoupanos**†

Physics Department, National Technical University
Zografou Campus, 157 80 Athens, Greece
E-mail: pman@central.ntua.gr, George.Zoupanos@cern.ch

Abstract: We address the question of supersymmetry breaking of a higher dimensional supersymmetric theory due to coset space dimensional reduction. In particular we study a ten-dimensional supersymmetric E_8 gauge theory which is reduced over all six-dimensional coset spaces. We find that the original supersymmetry is completely broken in the process of dimensional reduction when the coset spaces are symmetric. On the contrary softly broken four-dimensional supersymmetric theories result when the coset spaces are non-symmetric. From our analysis two promising cases are emerging which lead to interesting GUTs with three fermion families in four dimensions, one being non-supersymmetric and the other softly broken supersymmetric.

Keywords: Field Theories in Higher Dimensions, Supersymmetry Breaking, GUT, Compactification and String Models.

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1. Introduction

The celebrated Standard Model (SM) of Elementary Particle Physics had so far outstanding successes in all its confrontations with experimental results. However the apparent success of the SM is spoiled by the presence of a plethora of free parameters mostly related to the ad-hoc introduction of the Higgs and Yukawa sectors in the theory. It is worth recalling that the Coset Space Dimensional Reduction (CSDR) [1, 2, 3] was suggesting from the begining that a unification of the gauge and Higgs sectors can be achieved in higher dimensions. The four-dimensional gauge and Higgs fields are simply the surviving components of the gauge fields of a pure gauge theory defined in higher dimensions. In the next step of development of the CSDR scheme, fermions were introduced [4] and then the four-dimensional Yukawa and gauge interactions of fermions found also a unified description in the gauge interactions of the higher dimensional theory. The last step in this unified description in high dimensions is to relate the gauge and fermion fields that have been introduced. A simple way to achive that is to demand that the higher dimensional gauge theory is $\mathcal{N}=1$ supersymmetric which requires that the gauge and fermion fields are members of the same supermultiplet. An additional strong argument towards higher dimensional supersymmetry including gravity comes from the stability of the corresponding compactifying solutions that lead to the four-dimensional theory.

In the spirit described above a very welcome additional input is that string theory suggests furthermore the dimension and the gauge group of the higher dimensional supersymmetric theory [5]. Further support to this unified description comes from the fact that the reduction of the theory over coset [2] and CY spaces [5] provides the four dimensional theory with scalars belonging in the fundamental representation of the gauge group as are introduced in the SM. In addition the fact that the SM is a chiral theory lead us to consider D-dimensional supersymmetric gauge theories with $D = 4n + 2$ [6, 2], which include the ten dimensions suggested by the heterotic string theory [5].

Concerning supersymmetry, the nature of the four-dimensional theory depends on the corresponding nature of the compact space used to reduce the higher dimensional theory. Specifically the reduction over CY spaces leads to supersymmetric theories [5] in four dimensions, the reduction over symmetric coset spaces leads to non-supersymmetric theories, while a reduction over non-symmetric ones leads to softly broken supersymmetric theories [7]. Concerning the latter as candidate four-dimensional theories that describe the nature, in addition to the usual arguments related to the hierarchy problem [8], we should remind a further evidence established in their favor the last years. It was found that the search for renormalization group invariant (RGI) relations among parameters of softly broken supersymmetric GUTs considered as a unification scheme at the quantum level, could lead to successful predictions in low energies. More specifically the search for RGI relations was concerning the parameters of softly broken GUTs beyond the unification point and could lead even to all-loop finiteness [9, 10]. On the other hand in the low energies lead to successful predictions not only for the gauge couplings but also for the top quark mass, among others, and to interesting testable predictions for the Higgs mass [11].

The paper is organized as follows. In section 2 we present various details of the Coset Space Geometry with emphasis in the inclusion of torsion and more than one radii when possible. The CSDR scheme is also presented in sufficient detail to make the paper self-contained. In section 3 supersymmetry breaking via CSDR is examined. In 3.1 the issue of supersymmetry breaking in CSDR over symmetric coset spaces is analyzed with presentation of explicit examples of reduction of a supersymmetric ten-dimensional E_8 gauge theory over all symmetric six-dimensional coset spaces. Section 3.2 contains an explicit and detailed study of the supersymmetry breaking of the same supersymmetric ten-dimensional E_8 gauge theory, compactified over all non-symmetric coset spaces i.e. $G_2/\,\mathrm{SU}(3)$, $\mathrm{Sp}(4)/(\mathrm{SU}(2)\times \mathrm{U}(1))_{\mathrm{non-max}}$ and $\mathrm{SU}(3)/\,\mathrm{U}(1)\times \mathrm{U}(1)$. In section 4 we present our conclusions. The appendix A contains details related to the calculation of the gaugino mass in the reduction over the $\mathrm{SU}(3)/\,\mathrm{U}(1)\times \mathrm{U}(1)$ coset space.

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2. The coset space dimensional reduction

Given a gauge theory defined in higher dimensions the obvious way to dimensionally reduce it is to demand that the field dependence on the extra coordinates is such that the lagrangian is independent of them. A crude way to fulfill this requirement is to discard the field dependence on the extra coordinates, while an elegant one is to allow for a non-trivial dependence on them, but impose the condition that a symmetry transformation by an element of the isometry group S of the space formed by the extra dimensions B corresponds to a gauge transformation. Then the lagrangian will be independent of the extra coordinates just because it is gauge invariant. This is the basis of the CSDR scheme [1, 2, 3], which assumes that B is a compact coset space, S/R.

In the CSDR scheme one starts with a Yang-Mills-Dirac lagrangian, with gauge group G, defined on a D-dimensional spacetime M^D, with metric g^{MN}, which is compactified to $M^4 \times S/R$ with S/R a coset space. The metric is assumed to have the form

$$g^{MN} = \begin{bmatrix} \eta^{\mu\nu} & 0 \\ 0 & -g^{ab} \end{bmatrix}, \tag{2.1}$$

where $\eta^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ and g^{ab} is the coset space metric. The requirement that transformations of the fields under the action of the symmetry group of S/R are compensated by gauge transformations lead to certain constraints on the fields. The solution of these constraints provides us with the four-dimensional unconstrained fields as well as with the gauge invariance that remains in the theory after dimensional reduction. Therefore a potential unification of all low energy interactions, gauge, Yukawa and Higgs is achieved, which was the first motivation of this framework.

It is interesting to note that the fields obtained using the CSDR approach are the first terms in the expansion of the D-dimensional fields in harmonics of the internal space B. The effective field theories resulting from compactification of higher dimensional theories contain also towers of massive higher harmonics (Kaluza-Klein) excitations, whose contributions at the quantum level alter the behaviour of the running couplings from logarithmic to power [12]. As a result the traditional picture of unification of couplings may change drastically [13]. Higher dimensional theories have also been studied at the quantum level using the continuous Wilson renormalization group [14] which can be formulated in any number of space-time dimensions with results in agreement with the treatment involving massive Kaluza-Klein excitations.

Before we proceed with the description of the CSDR scheme we need to recall some facts about coset space geometry needed for subsequent discussions. Complete reviews can be found in [2, 15].

2.1 Coset space geometry

Assuming a D-dimensional spacetime M^D with metric g^{MN} given in eq. (2.1) it is instructive to explore further the geometry of all coset spaces S/R.

We can divide the generators of S, Q_A in two sets: the generators of R, Q_i ($i = 1, \ldots, \dim R$), and the generators of S/R, Q_a($a = \mathrm{dim}R + 1 \ldots, \mathrm{dim}S$), and $\mathrm{dim}S/R =$

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$\dim S - \dim R = d$. Then the commutation relations for the generators of S are the following:

$$\begin{aligned}
[Q_i, Q_j] &= f_{ij}{}^k Q_k\,, \\
[Q_i, Q_a] &= f_{ia}{}^b Q_b\,, \\
[Q_a, Q_b] &= f_{ab}{}^i Q_i + f_{ab}{}^c Q_c\,.
\end{aligned} \tag{2.2}$$

So S/R is assumed to be a reductive but in general non-symmetric coset space. When S/R is symmetric, the $f_{ab}{}^c$ in eq. (2.2) vanish. Let us call the coordinates of $M^4 \times S/R$ space $z^M = (x^m, y^\alpha)$, where α is a curved index of the coset, a is a tangent space index and y defines an element of S which is a coset representative, $L(y)$. The vielbein and the R-connection are defined through the Maurer-Cartan form which takes values in the Lie algebra of S:

$$L^{-1}(y) dL(y) = e^A_\alpha Q_A dy^\alpha\,. \tag{2.3}$$

Using eq. (2.3) we can compute that at the origin $y = 0$, $e^a_\alpha = \delta^a_\alpha$ and $e^i_\alpha = 0$. A connection on S/R which is described by a connection-form θ^a_b, has in general torsion and curvature. In the general case where torsion may be non-zero, we calculate first the torsionless part $\omega^a{}_b$ by setting the torsion form T^a equal to zero,

$$T^a = de^a + \omega^a{}_b \wedge e^b = 0\,, \tag{2.4}$$

while using the Maurer-Cartan equation,

$$de^a = \frac{1}{2} f^a{}_{bc} e^b \wedge e^c + f^a{}_{bi} e^b \wedge e^i\,, \tag{2.5}$$

we see that the condition of having vanishing torsion is solved by

$$\omega^a{}_b = -f^a{}_{ib} e^i - \frac{1}{2} f^a{}_{bc} e^c - \frac{1}{2} K^a{}_{bc} e^c\,, \tag{2.6}$$

where $K^a{}_{bc}$ is symmetric in the indices b, c, therefore $K^a{}_{bc} e^c \wedge e^b = 0$. The $K^a{}_{bc}$ can be found from the antisymmetry of $\omega^a{}_b$, $\omega^a{}_b g^{cb} = -\omega^b{}_c g^{ca}$, leading to

$$K^a{}_{bc} = g^{ad}(g_{be} f_{dc}{}^e + g_{ce} f_{db}{}^e)\,. \tag{2.7}$$

In turn $\omega^a{}_b$ becomes

$$\omega^a{}_b = -f^a{}_{ib} e^i - D^a{}_{bc} e^c\,, \tag{2.8}$$

where

$$D^a{}_{bc} = \frac{1}{2} g^{ad} [f_{db}{}^e g_{ec} + f_{cb}{}^e g_{de} - f_{cd}{}^e g_{be}]\,.$$

The D's can be related to f's by a rescaling [2]:

$$D^a{}_{bc} = (\lambda^a \lambda^b / \lambda^c) f^a{}_{bc}\,,$$

where the λ's depend on the coset radii. Note that in general the rescalings change the antisymmetry properties of f's, while in the case of equal radii $D^a{}_{bc} = \frac{1}{2} f^a{}_{bc}$. Note also that

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the connection-form is S-invariant. This means that parallel transport commutes with the S action [15]. Then the most general form of an S-invariant connection on S/R would be

$$\omega^a_{\ b} = f^a_{\ ib} e^i + J^a_{\ cb} e^c \,, \tag{2.9}$$

with J an R-invariant tensor, i.e.

$$\delta J_{cb}^{\ \ a} = -f_{ic}^{\ \ d} J_{db}^{\ \ a} + f_{id}^{\ \ a} J_{cb}^{\ \ d} - f_{ib}^{\ \ d} J_{cd}^{\ \ a} = 0 \,.$$

This condition is satisfied for D's as can be proven using the Jacobi idendity.

In the case of non-vanishing torsion we have

$$T^a = de^a + \theta^a_{\ b} \wedge e^b \,, \tag{2.10}$$

where

$$\theta^a_{\ b} = \omega^a_{\ b} + \tau^a_{\ b} \,,$$

with

$$\tau^a_{\ b} = -\frac{1}{2} \Sigma^a_{\ bc} e^c \,, \tag{2.11}$$

while the contorsion $\Sigma^a_{\ \ bc}$ is given by

$$\Sigma^a_{\ \ bc} = T^a_{\ \ bc} + T_{bc}^{\ \ a} - T_{cb}^{\ \ a} \tag{2.12}$$

in terms of the torsion components $T^a_{\ \ bc}$. Therefore in general the connection-form $\theta^a_{\ b}$ is

$$\theta^a_{\ b} = -f^a_{\ ib} e^i - \left(D^a_{\ bc} + \frac{1}{2} \Sigma^a_{\ bc} \right) e^c = -f^a_{\ ib} e^i - G^a_{\ bc} e^c \,. \tag{2.13}$$

The natural choice of torsion which would generalize the case of equal radii [16, 17, 18], $T^a_{\ bc} = \eta f^a_{\ bc}$ would be $T^a_{\ bc} = 2\tau D^a_{\ bc}$ except that the D's do not have the required symmetry properties. Therefore we must define Σ as a combination of D's which makes Σ completely antisymmetric and S-invariant according to the definition given above. Thus we are led to the definition

$$\Sigma_{abc} \equiv 2\tau (D_{abc} + D_{bca} - D_{cba}) \,. \tag{2.14}$$

In this general case the Riemann curvature two-form is given by [2, 18]:

$$R^a_{\ b} = \left[-\frac{1}{2} f_{ib}^{\ \ a} f_{de}^{\ \ i} - \frac{1}{2} G_{cb}^{\ \ a} f_{de}^{\ \ c} + \frac{1}{2} (G_{dc}^{\ \ a} G_{eb}^{\ \ c} - G_{ec}^{\ \ a} G_{db}^{\ \ c}) \right] e^d \wedge e^e \,, \tag{2.15}$$

whereas the Ricci tensor $R_{ab} = R^d_{\ adb}$ is

$$R_{ab} = G_{ba}^{\ \ c} G_{dc}^{\ \ d} - G_{bc}^{\ \ d} G_{da}^{\ \ c} - G_{ca}^{\ \ d} f_{db}^{\ \ c} - f_{ia}^{\ \ d} f_{db}^{\ \ i} \,. \tag{2.16}$$

By choosing the parameter τ to be equal to zero we can obtain the *riemannian connection* $\theta_{R\ \ b}^{\ a}$. We can also define the *canonical connection* by adjusting the radii and τ so that the connection form is $\theta_{C\ \ b}^{\ a} = -f^a_{\ bi} e^i$, i.e. an R-gauge field [16]. The adjustments should be such that $G_{abc} = 0$. In the case of $G_2/\,\mathrm{SU}(3)$ where the metric is $g_{ab} = a\delta_{ab}$, we have $G_{abc} = \frac{1}{2} a(1+3\tau) f_{abc}$ and in turn $\tau = -1/3$. In the case of $\mathrm{Sp}(4)/(\mathrm{SU}(2)\times\mathrm{U}(1))_{\mathrm{non-max}}$, where the metric is $g_{ab} = \mathrm{diag}(a,a,b,b,a,a)$, we have to set $a = b$ and then $\tau = -1/3$ to obtain the canonical connection. Similarly in the case of $\mathrm{SU}(3)/(\mathrm{U}(1)\times\mathrm{U}(1))$, where the metric is $g_{ab} = \mathrm{diag}(a,a,b,b,c,c)$, we should set $a = b = c$ and take $\tau = -1/3$. By analogous adjustments we can set the Ricci tensor equal to zero [16], thus defining a *Ricci flattening connection*.

2.2 Reduction of a D-dimensional Yang-Mills-Dirac lagrangian

The group S acts as a symmetry group on the extra coordinates. The CSDR scheme demands that an S-transformation of the extra d coordinates is a gauge transformation of the fields that are defined on $M^4 \times S/R$, thus a gauge invariant lagrangian written on this space is independent of the extra coordinates.

To see this in detail we consider a D-dimensional Yang-Mills-Dirac theory with gauge group G defined on a manifold M^D which as stated will be compactified to $M^4 \times S/R$, $D = 4 + d$, $d = \dim S - \dim R$:

$$A = \int d^4x d^d y \sqrt{-g} \left[-\frac{1}{4} \operatorname{Tr} \left(F_{MN} F_{K\Lambda} \right) g^{MK} g^{N\Lambda} + \frac{i}{2} \overline{\psi} \Gamma^M D_M \psi \right], \tag{2.17}$$

where

$$D_M = \partial_M + \theta_M - A_M \,, \tag{2.18}$$

with

$$\theta_M = \frac{1}{2} \theta_{MN\Lambda} \Sigma^{N\Lambda} \tag{2.19}$$

the spin connection of M^D, and

$$F_{MN} = \partial_M A_N - \partial_N A_M - [A_M, A_N] \,, \tag{2.20}$$

where M, N run over the D-dimensional space. The fields A_M and ψ are, as explained, symmetric in the sense that any transformation under symmetries of S/R is compensated by gauge transformations. The fermion fields can be in any representation F of G unless a further symmetry such as supersymmetry is required. So let ξ_A^α, $A = 1, \ldots, \dim S$, be the Killing vectors which generate the symmetries of S/R and W_A the compensating gauge transformation associated with ξ_A. Define next the infinitesimal coordinate transformation as $\delta_A \equiv L_{\xi_A}$, the Lie derivative with respect to ξ, then we have for the scalar,vector and spinor fields,

$$\begin{aligned} \delta_A \phi &= \xi_A^\alpha \partial_\alpha \phi = D(W_A) \phi \,, \\ \delta_A A_\alpha &= \xi_A^\beta \partial_\beta A_\alpha + \partial_\alpha \xi_A^\beta A_\beta = \partial_\alpha W_A - [W_A, A_\alpha] \,, \\ \delta_A \psi &= \xi_A^\alpha \psi - \frac{1}{2} G_{Abc} \Sigma^{bc} \psi = D(W_A) \psi \,. \end{aligned} \tag{2.21}$$

W_A depend only on internal coordinates y and $D(W_A)$ represents a gauge transformation in the appropriate representation of the fields. G_{Abc} represents a tangent space rotation of the spinor fields. The variations δ_A satisfy, $[\delta_A, \delta_B] = f_{AB}^C \delta_C$ and lead to the following consistency relation for W_A's,

$$\xi_A^\alpha \partial_\alpha W_B - \xi_B^\alpha \partial_\alpha W_A - [W_A, W_B] = f_{AB}^{\ \ C} W_C \,. \tag{2.22}$$

Furthermore the W's themselves transform under a gauge transformation [2] as,

$$\widetilde{W}_A = g W_A g^{-1} + (\delta_A g) g^{-1} \,. \tag{2.23}$$

Using eq. (2.23) and the fact that the lagrangian is independent of y we can do all calculations at $y = 0$ and choose a gauge where $W_a = 0$.

The detailed analysis of the constraints (2.21) given in refs. [1, 2] provides us with the four-dimensional unconstrained fields as well as with the gauge invariance that remains in the theory after dimensional reduction. Here we give the results. The components $A_\mu(x, y)$ of the initial gauge field $A_M(x, y)$ become, after dimensional reduction, the four-dimensional gauge fields and furthermore they are independent of y. In addition one can find that they have to commute with the elements of the R_G subgroup of G. Thus the four-dimensional gauge group H is the centralizer of R in G, $H = C_G(R_G)$. Similarly, the $A_\alpha(x, y)$ components of $A_M(x, y)$ denoted by $\phi_\alpha(x, y)$ from now on, become scalars at four dimensions. These fields transform under R as a vector v, i.e.

$$\begin{aligned} S &\supset R \\ \mathrm{adj}\, S &= \mathrm{adj}\, R + v\,. \end{aligned} \tag{2.24}$$

Moreover $\phi_\alpha(x, y)$ act as an intertwining operator connecting induced representations of R acting on G and S/R. This implies, exploiting Schur's lemma, that the transformation properties of the fields $\phi_\alpha(x, y)$ under H can be found if we express the adjoint representation of G in terms of $R_G \times H$:

$$\begin{aligned} G &\supset R_G \times H \\ \mathrm{adj}\, G &= (\mathrm{adj}\, R, 1) + (1, \mathrm{adj}\, H) + \sum (r_i, h_i)\,. \end{aligned} \tag{2.25}$$

Then if $v = \sum s_i$, where each s_i is an irreducible representation of R, there survives an h_i multiplet for every pair (r_i, s_i), where r_i and s_i are identical irreducible representations of R.

Turning next to the fermion fields [2, 4, 6, 19] similarly to scalars, they act as intertwining operators between induced representations acting on G and the tangent space of S/R, $\mathrm{SO}(d)$. Proceeding along similar lines as in the case of scalars to obtain the representation of H under which the four dimensional fermions transform, we have to decompose the representation F of the initial gauge group in which the fermions are assigned under $R_G \times H$, i.e.

$$F = \sum (t_i, h_i)\,, \tag{2.26}$$

and the spinor of $\mathrm{SO}(d)$ under R

$$\sigma_d = \sum \sigma_j\,. \tag{2.27}$$

Then for each pair t_i and σ_i, where t_i and σ_i are identical irreducible representations there is an h_i multiplet of spinor fields in the four-dimensional theory. In order however to obtain chiral fermions in the effective theory we have to impose further requirements. We first impose the Weyl condition in D dimensions. In $D = 4n + 2$ dimensions which is the case at hand, the decomposition of the left handed, say spinor under $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{SO}(d)$ is

$$\sigma_D = (2, 1, \sigma_d) + (1, 2, \overline{\sigma}_d)\,. \tag{2.28}$$

So we have in this case the decompositions

$$\sigma_d = \sum \sigma_k , \ \overline{\sigma}_d = \sum \overline{\sigma}_k \, . \tag{2.29}$$

Let us start from a vector-like representation F for the fermions. In this case each term (t_i, h_i) in eq. (2.26) will be either self-conjugate or it will have a partner $(\overline{t}_i, \overline{h}_i)$. According to the rule described in eqs. (2.26), (2.27) and considering σ_d we will have in four dimensions left-handed fermions transforming as $f_L = \sum h_k^L$. It is important to notice that since σ_d is non self-conjugate, f_L is non self-conjugate too. Similarly from $\overline{\sigma}_d$ we will obtain the right handed representation $f_R = \sum \overline{h}_k^R$ but as we have assumed that F is vector-like, $\overline{h}_k^R \sim h_k^L$. Therefore there will appear two sets of Weyl fermions with the same quantum numbers under H. This is already a chiral theory, but still one can go further and try to impose the Majorana condition in order to eliminate the doubling of the fermion spectrum. We should remark now that if we had started with F complex, we should have again a chiral theory since in this case $\overline{h}_k^R$ is different from h_k^L (σ_d non self-conjugate). Nevertheless starting with F vector-like is much more appealing and will be used in the following along with the Majorana condition. The Majorana condition can be imposed in $D = 2, 3, 4 + 8n$ dimensions and is given by $\psi = C\overline{\psi}^T$, where C is the D-dimensional charge conjugation matrix. Majorana and Weyl conditions are compatible in $D = 4n + 2$ dimensions. Then in our case if we start with Weyl-Majorana spinors in $D = 4n + 2$ dimensions we force f_R to be the charge conjugate to f_L, thus arriving in a theory with fermions only in f_L. Furthermore if F is to be real, then we have to have $D = 2 + 8n$, while for F pseudoreal $D = 6 + 8n$.

Starting with an anomaly free theory in higher dimensions, in ref. [20] was given the condition that has to be fulfilled in order to obtain anomaly free theories in four dimensions after dimensional reduction. The condition restricts the allowed embeddings of R into G [21, 2]. For $G = E_8$ in ten dimensions the condition takes the form

$$l(G) = 60 \, , \tag{2.30}$$

where $l(G)$ is the sum over all indices of the R representations appearing in the decomposition of the 248 representation of E_8 under $E_8 \supset R \times H$. The normalization is such that the vector representation in eq. (2.24) which defines the embedding of R into SO(6), has index two.

2.3 The four-dimensional theory

Next let us obtain the four-dimensional effective action. Assuming that the metric is block diagonal, taking into account all the constraints and integrating out the extra coordinates we obtain in four dimensions the following lagrangian:

$$A = C \int d^4x \left(-\frac{1}{4} F_{\mu\nu}^t F^{t\mu\nu} + \frac{1}{2} (D_\mu \phi_\alpha)^t (D^\mu \phi^\alpha)^t - V(\phi) + \frac{i}{2} \overline{\psi} \Gamma^\mu D_\mu \psi - \frac{i}{2} \overline{\psi} \Gamma^a D_a \psi \right) , \tag{2.31}$$

where $D_\mu = \partial_\mu - A_\mu$ and $D_a = \partial_a - \theta_a - \phi_a$ with $\theta_a = \frac{1}{2} \theta_{abc} \Sigma^{bc}$ the connection of the coset space, while C is the volume of the coset space. The potential $V(\phi)$ is given by:

$$V(\phi) = \frac{1}{4} g^{ac} g^{bd} \, \mathrm{Tr}(f_{ab}^C \phi_C - [\phi_a, \phi_b])(f_{cd}^D \phi_D - [\phi_c, \phi_d]) \, , \tag{2.32}$$

where, $A = 1, \ldots, \dim S$ and f 's are the structure constants appearing in the commutators of the generators of the Lie algebra of S. The expression (2.32)) for $V(\phi)$ is only formal because ϕ_a must satisfy the constraints coming from eq. (2.21),

$$f_{ai}^D \phi_D - [\phi_a, \phi_i] = 0\,, \tag{2.33}$$

where the ϕ_i generate R_G. These constraints imply that some components ϕ_a's are zero, some are constants and the rest can be identified with the genuine Higgs fields. When $V(\phi)$ is expressed in terms of the unconstrained independent Higgs fields, it remains a quartic polynomial which is invariant under gauge transformations of the final gauge group H, and its minimum determines the vacuum expectation values of the Higgs fields [22, 23]. The minimization of the potential is in general a difficult problem. If however S has an isomorphic image S_G in G which contains R_G in a consistent way then it is possible to allow the ϕ_a to become generators of S_G. That is $\overline{\phi}_a = \langle \phi^i \rangle Q_{ai} = Q_a$ with $\langle \phi^i \rangle Q_{ai}$ suitable combinations of G generators, Q_a a generator of S_G and a is also a coset-space index. Then

$$\begin{aligned} \overline{F}_{ab} &= f_{ab}{}^i Q_i + f_{ab}{}^c \overline{\phi}_c - [\overline{\phi}_a, \overline{\phi}_b] \\ &= f_{ab}{}^i Q_i + f_{ab}{}^c Q_c - [Q_a, Q_b] = 0\,, \end{aligned}$$

because of the commutation relations of S. Thus we have proven that $V(\phi = \overline{\phi}) = 0$ which furthermore is the minimum, because $V(\phi)$ is positive definite. Furthermore, the four-dimensional gauge group H breaks further by these non-zero vacuum expectation values of the Higgs fields to the centralizer K of the image of S in G, i.e. $K = C_G(S)$ [2, 22, 23]. This can been seen if we examine a gauge transformation of ϕ_a by an element h of H. Then we have

$$\phi_a \to h \phi_a h^{-1}, h \in H\,.$$

We note that the v.e.v. of the Higgs fields is gauge invariant for the set of h's that commute with S. That is h belongs to a subgroup K of H which is the centralizer of S_G in G.

More generally it can be proven [2] that dimensional reduction over a symmetric coset space always gives a potential of spontaneous breaking form. Note that in this case the potential acquires the form,

$$V(\phi) = \frac{1}{4} g^{ac} g^{bd}\, \mathrm{Tr}(f_{ab}{}^i J_i - [\phi_a, \phi_b])(f_{cd}{}^j J_j - [\phi_a, \phi_b])\,, \tag{2.34}$$

since the structure constants $f_{ab}{}^c$ are equal to zero. Next we decompose the adjoint representation of S under R,

$$\begin{aligned} S &\supset R \\ \mathrm{adj}\, S &= \mathrm{adj}\, R + \Sigma(s_a + \overline{s}_a)\,, \end{aligned} \tag{2.35}$$

and introduce the generators of the coset,

$$Q_S = (Q_i, Q_{s_a}, Q_{\overline{s}_a})\,, \tag{2.36}$$

where Q_i correspond to R and Q_{s_a} and $Q_{\overline{s}_a}$ to s_a and $\overline{s}_a$. With this notation and using the complex metric $g^{i\overline{j}}$ the potential (2.34) can be rewritten as

$$\begin{aligned} V(\phi) = & -\frac{1}{2} g^{s_a \overline{s}_a} g^{t_a \overline{t}_a} \operatorname{Tr}(f_{s_a t_a}{}^{i} J_i - [\phi_{s_a}, \phi_{t_a}])(f_{\overline{s}_a \overline{t}_a}{}^{j} J_j - [\phi_{\overline{s}_a}, \phi_{\overline{t}_a}]) - \\ & -\frac{1}{2} g^{s_a \overline{s}_a} g^{t_a \overline{t}_a} \operatorname{Tr}(f_{s_a \overline{t}_a}{}^{i} J_i - [\phi_{s_a}, \phi_{\overline{t}_a}])(f_{\overline{s}_a t_a}{}^{j} J_j - [\phi_{\overline{s}_a}, \phi_{t_a}]) . \end{aligned} \tag{2.37}$$

Note that the structure constants involved in the first and the second parentheses inside the traces in eq. (2.37) are of opposite sign, since they appear in the commutator of conjugate generators. The same is true for the commutator of two ϕ fields, since they are actually expressed as linear combinations of the gauge group generators; if ϕ_{s_a} is connected to one generator, then $\phi_{\overline{s}_a}$ will be connected to its conjugate generator. As a result, terms involving two J_i will be constant positive terms, terms with one J_i and a ϕ commutator will be negative mass terms, and finally terms involving two ϕ commutators will be quartic positive terms. This result remains unaltered if the more general case is considered, where the vector of the coset S/R decomposed under R contains also real representations. So in conclusion the potential obtained from the dimensional reduction of a gauge theory over symmetric coset spaces is always of a spontaneous breaking form.

In the fermion part of the lagrangian the first term is just the kinetic term of fermions, while the second is the Yukawa term [24]. Note that since ψ is a Majorana-Weyl spinor in ten dimensions the representation in which the fermions are assigned under the gauge group must be real. The last term in eq. (2.31) can be written as

$$L_Y = -\frac{i}{2} \overline{\psi} \Gamma^a \left(\partial_a - \frac{1}{2} f_{ibc} e^i_\gamma e^\gamma_a \Sigma^{bc} - \frac{1}{2} G_{abc} \Sigma^{bc} - \phi_a \right) \psi = \frac{i}{2} \overline{\psi} \Gamma^a \nabla_a \psi + \overline{\psi} V \psi , \tag{2.38}$$

where

$$\begin{aligned} \nabla_a &= -\partial_a + \frac{1}{2} f_{ibc} e^i_\gamma e^\gamma_a \Sigma^{bc} + \phi_a , \\ V &= \frac{i}{4} \Gamma^a G_{abc} \Sigma^{bc} , \end{aligned} \tag{2.39}$$

and we have used the full connection with torsion [18] given by

$$\theta^a{}_{cb} = -f^a{}_{ib} e^i_\alpha e^\alpha_c - \left(D^a{}_{cb} + \frac{1}{2} \Sigma^a{}_{cb} \right) = -f^a{}_{ib} e^i_\alpha e^\alpha_c - G^a{}_{cb} \tag{2.40}$$

with

$$D^a{}_{cb} = g^{ad} \frac{1}{2} [f_{db}{}^e g_{ec} + f_{cb}{}^e g_{de} - f_{cd}{}^e g_{be}] \tag{2.41}$$

and

$$\Sigma_{abc} = 2\tau (D_{abc} + D_{bca} - D_{cba}) . \tag{2.42}$$

We have already noticed that the CSDR constraints tell us that $\partial_a \psi = 0$. Furthermore we can consider the lagrangian at the point $y = 0$, due to its invariance under S-transformations, and as we mentioned $e^i_\gamma = 0$ at that point. Therefore in eq. (2.39) we obtain just $\nabla_a = \phi_a$ and the term $(i/2)\overline{\psi}\Gamma^a \nabla_a \psi$ in eq. (2.38) is exactly the Yukawa term.

Let us examine now the last term appearing in eq. (2.38). One can show easily that the operator V anticommutes with the six-dimensional helicity operator [2]. Furthermore one can show that V commutes with the $T_i = (-1/2) f_{ibc}\Sigma^{bc}$ (T_i close the R-subalgebra of SO(6)). In turn we can draw the conclusion, exploiting Schur's lemma, that the non-vanishing elements of V are only those which appear in the decomposition of both SO(6) irreps 4 and $\overline{4}$, e.g. the singlets. Since this term is of pure geometric nature, we reach the conclusion that the singlets in 4 and $\overline{4}$ will acquire large geometrical masses, a fact that has serious phenomenological implications. In supersymmetric theories defined in higher dimensions, it means that the gauginos obtained in four dimensions after dimensional reduction receive masses comparable to the compactification scale. However as we shall see in the next sections this result changes in presence of torsion. We note that for symmetric coset spaces the V operator is absent because f_{ab}^c are vanishing by definition in that case.

3. Supersymmetry breaking by dimensional reduction over coset spaces

Recently a lot of interest has been triggered by the possibility that superstrings can be defined at the TeV scale [25]. The string tension became an arbitrary parameter and can be anywhere below the Planck scale and as low as TeV. The main advantage of having the string tension at the TeV scale, besides the obvious experimental interest, is that it offers an automatic protection to the gauge hierarchy [25], alternative to low energy supersymmetry [8], or dynamical electroweak symmetry breaking [26, 27, 28]. However the only vacua of string theory free of all pathologies are supersymmetric. Then the original supersymmetry of the theory, not being necessary in four dimensions, could be broken by the dimensional reduction procedure.

The weakly coupled ten-dimensional $E_8 \times E_8$ supersymmetric gauge theory is one of the few to posses the advantage of anomaly freedom [29] and has been extensively used in efforts to describe quantum gravity along with the observed low energy interactions in the heterotic string framework [5]. In addition its strong coupling limit provides an interesting example of the realization of the brane picture, i.e. E_8 gauge fields and matter live on the two 10-dimensional boundaries, while gravitons propagate in the eleven-dimensional bulk [30].

In the following we shall be reducing a supersymmetric ten-dimensional gauge theory based on E_8 over coset spaces and examine the consequences of the resulting four-dimensional theory mostly as far as supersymmetry breaking is concerned.

3.1 Supersymmetry breaking by dimensional reduction over symmetric coset spaces

Let us first examine the reduction of the ten-dimensional $\mathcal{N} = 1$ supersymmetric E_8 gauge theory over the symmetric coset spaces of table 1.

3.1.1 Reduction of $G = E_8$ over $B = \mathrm{SO}(7)/\mathrm{SO}(6)$

First we review the reduction of E_8 over the 6-sphere [2, 6, 7]. In that case $B = \mathrm{SO}(7)/\mathrm{SO}(6)$, $D = 10$ and the Weyl-Majorana fermions belong in the adjoint of G. The embedding

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S/R	SO(6) vector	SO(6) spinor
$\mathrm{SO}(7)/\mathrm{SO}(6)$	6	4
$\mathrm{SU}(4)/\mathrm{SU}(3)\times\mathrm{U}(1)$	$3_{-2}+\overline{3}_2$	1_3+3_{-1}
$\mathrm{Sp}(4)/(\mathrm{SU}(2)\times\mathrm{U}(1))_{\max}$	$3_{-2}+3_2$	1_3+3_{-1}
$\mathrm{SU}(3)\times\mathrm{SU}(2)/$ $\mathrm{SU}(2)\times\mathrm{U}(1)\times\mathrm{U}(1)$	$1_{0,2a}+1_{0,-2a}+$ $+2_{b,0}+2_{-b,0}$	$1_{b,-a}+1_{-b,-a}+$ $+2_{0,a}$
$\mathrm{Sp}(4)\times\mathrm{SU}(2)/$ $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$	$(1,1)_2+(1,1)_{-2}+$ $+(2,2)_0$	$(2,1)_1+(1,2)_{-1}$
$(\mathrm{SU}(2)/\mathrm{U}(1))^3$	$(2a,0,0)+(-2a,0,0)+$ $+(0,2b,0)+(0,-2b,0)+$ $+(0,0,2c)+(0,0,-2c)$	$(a,b,c)+(-a,-b,c)+$ $+(-a,b,-c)+(a,-b,-c)$

Table 1: Six-dimensional symmetric cosets with $\mathrm{rank}S = \mathrm{rank}R$.

of $R = \mathrm{SO}(6)$ in E_8 is suggested by the decomposition

$$\begin{aligned} E_8 &\supset \mathrm{SO}(6)\times\mathrm{SO}(10) \\ 248 &= (15,1)+(1,45)+(6,10)+(4,16)+(\overline{4},\overline{16})\,. \end{aligned} \tag{3.1}$$

The $R = \mathrm{SO}(6)$ content of the vector and spinor of $\mathrm{SO}(7)/\mathrm{SO}(6)$ are 6 and 4 respectively. The condition that guarantees the anomaly freedom of the four-dimensional theory given in eq. (2.30) is satisfied and the resulting gauge group is $H = C_{E_8}(\mathrm{SO}(6)) = \mathrm{SO}(10)$. According to the CSDR rules (2.24), (2.25) and (2.26), (2.27) the surviving scalars in four dimensions transform as a 10-plet under the gauge group SO(10), while the surviving fermions provide the four-dimensional theory with a 16_L and a $\overline{16}_R$ which are identified by the Weyl-Majorana condition. Concerning supersymmetry obviously any sign of the supersymmetry of the original theory has disappeared in the process of dimensional reduction. On the other hand the four dimensional theory is a GUT with fermions in a multiplet which is appropriate to describe quarks and leptons (including a right-handed neutrino). Finally since the SO(7) has an isomorphic image in E_8, according to the theorem discussed in the subsection 2.3, the SO(10) breaks further due to the v.e.v. of the 10-plet Higgs down to $C_{E_8}(\mathrm{SO}(7)) = \mathrm{SO}(9)$. Therefore the scalar field content of the four-dimensional theory is appropriate for the electroweak symmetry breaking but not for the GUT breaking.

3.1.2 Reduction of $G = E_8$ over $B = \mathrm{SU}(4)/\mathrm{SU}(3)\times\mathrm{U}(1)$

In this case $G = E_8$ and $S/R = \mathrm{SU}(4)/\mathrm{SU}(3)\times\mathrm{U}(1)$. The embedding of $R = \mathrm{SU}(3)\times\mathrm{U}(1)$ is determined by the following decomposition

$$\begin{aligned} E_8 &\supset \mathrm{SU}(3)\times\mathrm{U}(1)\times\mathrm{SO}(10) \\ 248 &= (1_0+3_{-4}+\overline{3}_4+8_0,1)+(1_0,45)+(3_2+\overline{3}_{-2},10)+(1_3+3_{-1},16)+(1_{-3}+3_1,\overline{16})\,, \end{aligned} \tag{3.2}$$

where the $\mathrm{SU}(3)\times\mathrm{U}(1)$ is the maximal subgroup of $\mathrm{SO}(6)\approx\mathrm{SU}(4)$ appearing in the decomposition (3.2). The R is chosen to be identified with the $\mathrm{SU}(3)\times\mathrm{U}(1)$ of the above decomposition. Therefore the resulting four-dimensional gauge group is $H = C_{E_8}(\mathrm{SU}(3) \times \mathrm{U}(1)) = \mathrm{SO}(10) \times \mathrm{U}(1)$ (The U(1) appears since U(1) in R centralizes itself). The $R = \mathrm{SU}(3) \times \mathrm{U}(1)$ content of the $\mathrm{SU}(4)/\mathrm{SU}(3) \times \mathrm{U}(1)$ vector and spinor can be read from table 1 and are $3_{-2} + \overline{3}_2$ and $1_3 + 3_{-1}$ respectively. Therefore we find that the surviving scalars in four dimensions transform as 10_2 and 10_{-2}, while the four-dimensional fermions transform as 16_3 and 16_{-1} under the four-dimensional gauge group $H = \mathrm{SO}(10) \times \mathrm{U}(1)$. Again there is no sign of the original supersymmetry in four dimensions.

3.1.3 Reduction of $G = E_8$ over $B = \mathrm{Sp}(4)/(\mathrm{SU}(2) \times \mathrm{U}(1))_{\max}$

Next we choose $G = E_8$ and $S/R = \mathrm{Sp}(4)/(\mathrm{SU}(2) \times \mathrm{U}(1))_{\max}$. The embedding of $R = (\mathrm{SU}(2) \times \mathrm{U}(1))_{\max}$ is determined by the decomposition (3.2) when the SU(2) is chosen to be the maximal subgroup of SU(3). In that case the decomposition of the 248 of E_8 is the following

$$\begin{aligned} E_8 &\supset \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{SO}(10) \\ 248 &= (1,1)_0 + (3,1)_{-4} + (3,1)_4 + (3,1)_0 + (5,1)_0 + (1,45)_0 + (3,10)_2 + (3,10)_{-2} + \\ &\quad +(1,16)_3 + (3,16)_{-1} + (1,\overline{16})_{-3} + (3,\overline{16})_1 \,. \end{aligned} \tag{3.3}$$

From the decomposition (3.3) is clear that the four-dimensional gauge group is $H = C_{E_8}((\mathrm{SU}(2) \times \mathrm{U}(1))_{\max}) = \mathrm{SO}(10) \times \mathrm{U}(1)$. The $R = (\mathrm{SU}(2) \times \mathrm{U}(1))_{\max}$ content of the $\mathrm{Sp}(4)/(\mathrm{SU}(2) \times \mathrm{U}(1))_{\max}$ vector and spinor according to table 1 are $3_{-2} + 3_2$ and $1_3 + 3_{-1}$ respectively under R. Therefore the particle content of the four dimensional theory is a set of 10_2, 10_{-2} scalars and a set of 16_3, 16_{-1} left handed spinors. Once more no sign of the original supersymmetry is left in the spectrum of the four-dimensional theory.

3.1.4 Reduction of $G = E_8$ over $B = \mathrm{Sp}(4) \times \mathrm{SU}(2)/\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$

Next we choose again $G = E_8$ while $S/R = \mathrm{Sp}(4) \times \mathrm{SU}(2)/\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. The embedding of $R = \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ is given by the decomposition

$$\begin{aligned} E_8 &\supset \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{SO}(10) \\ 248 &= (3,1,1)_0 + (1,3,1)_0 + (1,2,16)_{-1} + (1,2,\overline{16})_1 + (1,1,1)_0 + (1,1,45)_0 + \\ &\quad +(1,1,10)_2 + (1,1,10)_{-2} + (2,2,1)_2 + (2,2,1)_{-2} + (2,2,10)_0 + \\ &\quad +(2,1,16)_1 + (2,1,\overline{16})_{-1} \,, \end{aligned} \tag{3.4}$$

where the $R = \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ is the maximal subgroup of $\mathrm{SO}(6) \approx \mathrm{SU}(4)$ appearing in the decomposition (3.1). The four dimensional gauge group that survives after dimensional reduction is $H = C_{E_8}(\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)) = \mathrm{SO}(10) \times \mathrm{U}(1)$. According to table 1 the $R = \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ content of the $\mathrm{Sp}(4) \times \mathrm{SU}(2)/\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ vector and spinor are $(2,2)_0 + (1,1)_2 + (1,1)_{-2}$ and $(2,1)_1 + (1,2)_{-1}$, respectively. Therefore the scalar fields that survive in four dimensions belong to 10_0, 10_2, 10_{-2} of $H = \mathrm{SO}(10) \times \mathrm{U}(1)$. Similarly the surviving fermions in four dimensions transform as 16_1, 16_{-1} left-handed multiplets.

3.1.5 Reduction of $G = E_8$ over $B = \mathrm{SU}(3) \times \mathrm{SU}(2)/\,\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$

Choosing $G = E_8$ and $S/R = \mathrm{SU}(3) \times \mathrm{SU}(2)/\,\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$ we have another interesting example. The embedding of $R = \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$ in E_8 is given by the decomposition

$$\begin{aligned} E_8 &\supset \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1) \times \mathrm{SO}(10) \\ 248 &= (1,45)_{(0,0)} + (3,1)_{(0,0)} + (1,1)_{(0,0)} + (1,1)_{(0,0)} + (1,1)_{(2,0)} + (1,1)_{(-2,0)} + \\ &\quad +(2,1)_{(1,2)} + (2,1)_{(-1,2)} + (2,1)_{(-1,-2)} + (2,1)_{(1,-2)} + (1,10)_{(0,2)} + (1,10)_{(0,-2)} + \\ &\quad +(2,10)_{(1,0)} + (2,10)_{(-1,0)} + (2,16)_{(0,1)} + (1,16)_{(1,-1)} + (1,16)_{(-1,-1)} + \\ &\quad +(2,\overline{16})_{(0,-1)} + (1,\overline{16})_{(-1,1)} + (1,\overline{16})_{(1,1)}\,, \end{aligned} \tag{3.5}$$

where the $R = \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$ is identified with the one appearing in the following decomposition of maximal subgroups

$$\mathrm{SO}(6) \supset \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1) \supset \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$$

and the $R = \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ in E_8 is the maximal subgroup of $\mathrm{SO}(6)$ appearing in the decomposition (3.1). We find that the four-dimensional gauge group is $H = C_{E_8}(\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)) = \mathrm{SO}(10) \times \mathrm{U}(1) \times \mathrm{U}(1)$. The vector and spinor content under R of the specific coset can be found in table 1. Choosing $a = b = 1$ we find that the scalar fields of the four-dimensional theory transform as $10_{(0,2)}$, $10_{(0,-2)}$, $10_{(1,0)}$, $10_{(-1,0)}$ under H. Also, we find that the fermions of the four-dimensional theory are the following left-handed multiplets of H: $16_{(-1,-1)}$, $16_{(1,-1)}$, $16_{(0,1)}$.

3.1.6 Reduction of $G = E_8$ over $B = (\mathrm{SU}(2)/\,\mathrm{U}(1))^3$

Last we examine the case with $G = E_8$ and $S/R = (\mathrm{SU}(2)/\,\mathrm{U}(1))^3$. The $R = \mathrm{U}(1)^3$ is chosen to be identified with the three $\mathrm{U}(1)$ subgroups of $\mathrm{SO}(6)$ appearing in the decomposition

$$\mathrm{SO}(6) \supset \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1) \supset \mathrm{U}(1) \times \mathrm{U}(1) \times \mathrm{U}(1)\,,$$

where the $\mathrm{SO}(6)$ is again the one of the decomposition (3.1). Then we find the following decomposition of 248 of E_8,

$$\begin{aligned} E_8 &\supset \mathrm{U}(1) \times \mathrm{U}(1) \times \mathrm{U}(1) \times \mathrm{SO}(10) \\ 248 &= 1_{(0,0,0)} + 1_{(0,0,0)} + 1_{(0,0,0)} + 1_{(0,0,\pm 2)} + 1_{(\pm 4,\mp 2,0)} + \\ &\quad +1_{(0,\pm 3,\pm 1)} + 1_{(0,\mp 3,\pm 1)} + 1_{(\pm 4,\pm 1,\pm 1)} + 1_{(\pm 4,\pm 1,\mp 1)} + \\ &\quad +45_{(0,0,0)} + 16_{(-3,0,0)} + \overline{16}_{(3,0,0)} + 16_{(1,-2,0)} + \\ &\quad +\overline{16}_{(-1,2,0)} + 10_{(-2,-2,0)} + \overline{10}_{(2,2,0)} + \\ &\quad +16_{(1,1,1)} + \overline{16}_{(-1,-1,-1)} + 16_{(1,1,-1)} + \overline{16}_{(-1,-1,1)} + \\ &\quad +10_{(-2,1,1)} + \overline{10}_{(2,-1,-1)} + \\ &\quad +10_{(-2,1,-1)} + \overline{10}_{(2,-1,1)}\,. \end{aligned} \tag{3.6}$$

Therefore the four-dimensional gauge group is $H = C_{E_8}(\mathrm{U}(1)^3) = \mathrm{SO}(10) \times \mathrm{U}(1)^3$. The $R = \mathrm{U}(1)^3$ content of $(\mathrm{SU}(2)/\,\mathrm{U}(1))^3$ vector and spinor can be found in table 1. With

S/R	SO(6) vector	SO(6) spinor
$G_2/\,\mathrm{SU}(3)$	$3+\overline{3}$	$1+3$
$\mathrm{Sp}(4)/(\mathrm{SU}(2)\times \mathrm{U}(1))_{\mathrm{non-max}}$	$1_2+1_{-2}+2_1+2_{-1}$	$1_0+1_2+2_{-1}$
$\mathrm{SU}(3)/\,\mathrm{U}(1)\times\mathrm{U}(1)$	$(a,c)+(b,d)+(a+b,c+d)+$ $+(-a,-c)+(-b,-d)+$ $+(-a-b,-c-d)$	$(0,0)+(a,c)+(b,d)+$ $+(-a-b,-c-d)$

Table 2: Six-dimensional non-symmetric cosets with $rankS = rankR$.

$a = b = c = 1$ the vector and spinor become $(0,0,\pm 2)+(\pm 2,0,0)+(0,\pm 2,0)$ and $(\pm 1,\pm 1,1)+(\mp 1,\pm 1,-1)$ respectively and therefore the four-dimensional scalar fields transform as $10_{(0,0,2)}$, $10_{(0,0,-2)}$, while fermions transform as left-handed $16_{(1,1,1)}$, $16_{(1,-1,-1)}$ under H.

Note that in all above cases 3.1.1–3.1.6 the chosen embedding of R in G satisfy eq. (2.30) and therefore the resulting four-dimensional theories are anomaly free.

3.2 Supersymmetry breaking by dimensional reduction over non-symmetric coset spaces

Next we start with the same theory in ten dimensions but we reduce it over the three non-symmetric coset spaces listed in table 2.

3.2.1 Soft supersymmetry breaking by dimensional reduction over $G_2/\,\mathrm{SU}(3)$

First we choose $B = G_2/\,\mathrm{SU}(3)$ [7]. We use the decomposition

$$\begin{aligned} E_8 &\supset \mathrm{SU}(3)\times E_6 \\ 248 &= (8,1)+(1,78)+(3,27)+(\overline{3},\overline{27}) \end{aligned} \tag{3.7}$$

and we choose SU(3) to be identified with R. The $R = \mathrm{SU}(3)$ content of the $G_2/\,\mathrm{SU}(3)$ vector and spinor is $3+\overline{3}$ and $1+3$ as can be read from table 2. The condition (2.30) for the cancellation of anomalies is satisfied and the resulting four dimensional gauge group is $H = C_{E_8}(\mathrm{SU}(3)) = E_6$, which contains fermion and complex scalar fields transforming as 78, 27 and 27 respectively. Therefore we obtain in four dimensions a $\mathcal{N}=1$ supersymmetric anomaly free E_6 gauge theory with a vector superfield grouping gauge bosons and fermions transforming according to the adjoint and a matter chiral superfield grouping scalars and fermions in the fundamental of the gauge group E_6. In addition a very interesting feature worth stressing is that the $\mathcal{N}=1$ supersymmetry of the four-dimensional theory is broken by soft terms. More precisely the scalar soft terms appear in the potential of the theory and the gaugino masses come from a geometric (torsion) term as already stated.

We proceed by calculating these terms. In order to determine the potential we begin by examining the decomposition of the specific S under R, i.e.

$$\begin{aligned} G_2 &\supset \mathrm{SU}(3) \\ 14 &= 8+3+\overline{3}\,. \end{aligned} \tag{3.8}$$

$[Q^a, Q^b] = 2if^{abc}Q^c$	$[Q^a, Q^\rho] = -(\lambda^a)^\rho_\sigma Q^\sigma$
$[Q^\rho, Q_\sigma] = -(\lambda^a)^\rho_\sigma Q^a$	$[Q^\rho, Q^\sigma] = 2\sqrt{2/3}\epsilon^{\rho\sigma\tau}Q_\tau$

Table 3: Non-trivial commutation relations of G_2 according to the decomposition given in eq. (3.9). The normalization is $\operatorname{Tr} Q^a Q^b = 2\delta^{ab}$, $\operatorname{Tr} Q^\rho Q_\sigma = 2\delta^\rho_\sigma$.

$[Q^a, Q^b] = 2if^{abc}Q^c$	$[Q^\alpha, Q^\beta] = 2ig^{\alpha\beta\gamma}Q^\gamma$
$[Q^a, Q^{i\rho}] = -(\lambda^a)^\rho_\sigma \delta^i_j Q^{j\sigma}$	$[Q^{i\rho}, Q^{j\sigma}] = \frac{1}{\sqrt{6}}\epsilon^{\rho\sigma\tau}d^{ijk}Q_{k\tau}$
$[Q^{i\rho}, Q_{j\sigma}] = -(\lambda^a)^\rho_\sigma \delta^i_j Q^a +$ $\frac{1}{6}\delta^\rho_\sigma (G^\alpha)^i_j Q^\alpha$	$[Q^\alpha, Q^{i\rho}] = (G^\alpha)^i_j \delta^\rho_\sigma Q^{j\sigma}$

Table 4: Non-trivial commutation relations of E_8 according to the decomposition given in eq. (3.12). The normalization is $\operatorname{Tr} Q^a Q^b = 2\delta^{ab}$, $\operatorname{Tr} Q^\alpha Q^\beta = 12\delta^{\alpha\beta}$, $\operatorname{Tr} Q^{i\rho} Q_{j\sigma} = 2\delta^i_j\delta^\rho_\sigma$.

Corresponding to this decomposition we introduce the generators of G_2

$$Q_{G_2} = \{Q^a, Q^\rho, Q_\rho\}\,, \tag{3.9}$$

where $a = 1, \ldots, 8$ correspond to the 8 of SU(3), while $\rho = 1, 2, 3$ correspond to 3 or $\overline{3}$. Then according to the decomposition (3.9), the non trivial commutation relations of the generators of G_2 are given in table 3.

The potential of any theory reduced over $G_2/\mathrm{SU}(3)$ can be written in terms of the fields

$$\{\phi^a, \phi^\rho, \phi_\rho\}, \tag{3.10}$$

which correspond to the decomposition (3.9) of G_2 under SU(3). The ϕ_a are equal to the generators of the R subgroup. With the help of the commutation relations of table 3 we find that the potential of any theory reduced over $G_2/\mathrm{SU}(3)$ is given by [2]:

$$\begin{aligned} V(\phi) = {} & \frac{8}{R_1^4} + \frac{4}{3R_1^4}\operatorname{Tr}(\phi^\rho\phi_\rho) - \frac{1}{2R_1^4}(\lambda^i)^\rho_\sigma \operatorname{Tr}(J_i[\phi_\rho, \phi^\sigma]) + \\ & + \left[\frac{1}{R_1^4}\sqrt{\frac{2}{3}}\epsilon^{\rho\sigma\tau}\operatorname{Tr}(\phi_\tau[\phi_\rho, \phi_\sigma]) + \text{h.c.}\right] - \frac{1}{4R_1^4}\operatorname{Tr}([\phi_\rho, \phi_\sigma][\phi^\rho, \phi^\sigma] + [\phi^\rho, \phi_\sigma][\phi_\rho, \phi^\sigma])\,, \end{aligned} \tag{3.11}$$

where the R_1 appearing in the denominator of various terms is the radius of $G_2/\mathrm{SU}(3)$. Then to proceed with our specific choice $G = E_8$ we use the embedding (3.7) of $R = \mathrm{SU}(3)$ in E_8 and divide accordingly the generators of E_8

$$Q_{E_8} = \{Q^a, Q^\alpha, Q^{i\rho}, Q_{i\rho}\} \tag{3.12}$$

with $a = 1, \ldots, 8$, $\alpha = 1, \ldots, 78$, $i = 1, \ldots, 27$, $\rho = 1, 2, 3$. The non-trivial commutation relations of the generators of E_8 according to the decomposition (3.12) are given in table 4. Next we would like to solve the constraints (2.33) which in the present case take the form $[\phi^a, \phi^\rho] = -(\lambda^a)^\rho_\sigma\phi^\sigma$, and examine the resulting four-dimensional potential in terms of the unconstrained scalar fields β. The solutions of the constraints in terms of the genuine Higgs fields are

$$\phi^a = Q^a,\ \phi_\rho = R_1\beta^i Q_{i\rho},\ \phi^\rho = R_1\beta_i Q^{i\rho}\,. \tag{3.13}$$

In turn we can express the Higgs potential in terms of the genuine Higgs field β and we find

$$V(\beta) = \frac{8}{R_1^4} - \frac{40}{3R_1^2}\beta^2 - \left[\frac{4}{R_1} d_{ijk}\beta^i\beta^j\beta^k + \text{h.c.}\right] + \beta^i\beta^j d_{ijk}d^{klm}\beta_l\beta_m + \\ + \frac{11}{4}\sum_\alpha \beta^i (G^\alpha)_i^j \beta_j \beta^k (G^\alpha)_k^l \beta_l\,, \tag{3.14}$$

where d^{ijk}, the symmetric invariant E_6 tensor, and $(G^\alpha)_j^i$ are defined in ref. [31]. From the potential given in eq. (3.14) we can read directly the F-, D- and scalar soft terms which break softly the supersymmetric theory obtained by CSDR over $G_2/\,\mathrm{SU}(3)$. The F-terms are obtained from the superpotential

$$\mathcal{W}(B) = \frac{1}{3} d_{ijk} B^i B^j B^k, \tag{3.15}$$

where B is the chiral superfield whose scalar component is β. Let us note that the superpotential could also be identified from the relevant Yukawa terms of the fermion part of the lagrangian. Correspondingly the D-terms are

$$D^\alpha = \sqrt{\frac{11}{2}}\beta^i (G^\alpha)_i^j \beta_j. \tag{3.16}$$

The terms in the potential $V(\beta)$ given in eq. (3.14) that do not result from the F- and D-terms belong to the soft supersymmetry part of the lagrangian. These terms are the following

$$\mathcal{L}_{\text{scalarSSB}} = -\frac{40}{3R_1^2}\beta^2 - \left[\frac{4}{R_1} d_{ijk}\beta^i\beta^j\beta^k + \text{h.c.}\right]. \tag{3.17}$$

Note that the potential (3.14) belongs to the case, discussed in subsection 2.3, that S can be embedded in G [32]. Finally in order to determine the gaugino mass we calculate the V operator given in eq. (2.39). Using eq. (2.41) we find that

$$D_{abc} = \frac{R_1^2}{2} f_{abc} \tag{3.18}$$

and in turn the $G_{abc} = D_{abc} + \frac{1}{2}T_{abc}$ is

$$G_{abc} = \frac{R_1^2}{2}(1+3\tau) f_{abc}\,. \tag{3.19}$$

In order to obtain the previous results the most general G_2 invariant metric on $G_2/\,\mathrm{SU}(3)$ was used which is $g_{ab} = R_1^2\delta_{ab}$. In addition we need the gamma matrices in ten dimensions given in appendix A. We find that the gauginos acquire a geometrical mass

$$M = (1+3\tau)\frac{6}{\sqrt{3}R_1}\,. \tag{3.20}$$

$[Q_\rho, Q_\sigma] = 2i\epsilon_{\rho\sigma\tau} Q_\tau$	$[Q, Q_a] = Q_a$	$[Q_\rho, Q_a] = (\tau_\rho)_a^b Q_b$
$[Q, Q_+] = 2Q_+$	$[Q_a, Q^+] = -\sqrt{2}\epsilon_{ab} Q^b$	$[Q_a, Q_b] = \sqrt{2}\epsilon_{ab} Q_+$
$\left[Q_a, Q^b\right] = \delta_b^a Q + (\tau_\rho)_a^b Q_\rho$	$[Q_+, Q^+] = 2Q$	

Table 5: Non-trivial commutation relations of Sp(4) according to the decomposition given in eq. (3.23). The normalization is given by $\mathrm{Tr}(Q_\rho Q_\sigma) = 2\delta_{\rho\sigma}$, $\mathrm{Tr}(Q_a Q^b) = 2\delta_a^b$, $\mathrm{Tr}(Q_+ Q^+) = 2$.

3.2.2 Soft supersymmerty breaking by dimensional reduction over $\mathrm{Sp}(4)/(\mathrm{SU}(2) \times \mathrm{U}(1))_{\mathrm{non-max}}$

In this case we start again with a ten-dimensional supersymmetric gauge theory based on the group E_8 and reduce it over the non-symmetric coset $\mathrm{Sp}(4)/(\mathrm{SU}(2) \times \mathrm{U}(1))_{\mathrm{non-max}}$. Therefore we have chosen $G = E_8$ and $B = \mathrm{Sp}(4)/(\mathrm{SU}(2) \times \mathrm{U}(1))_{\mathrm{non-max}}$. We begin by giving the decompositions to be used,

$$E_8 \supset \mathrm{SU}(3) \times E_6 \supset \mathrm{SU}(2) \times \mathrm{U}(1) \times E_6 \,.$$

The decomposition of 248 of E_8 under $\mathrm{SU}(3) \times E_6$ is given in eq. (3.7) while under $(\mathrm{SU}(2) \times \mathrm{U}(1)) \times E_6$ is the following:

$$\begin{aligned} 248 \,=\, & (3,1)_0 + (1,1)_0 + (1,78)_0 + (2,1)_3 + (2,1)_{-3} + \\ & +(1,27)_{-2} + (1,\overline{27})_2 + (2,27)_1 + (2,\overline{27})_{-1} \,. \end{aligned} \tag{3.21}$$

In the present case R is chosen to be identified with the $\mathrm{SU}(2) \times \mathrm{U}(1)$ of the latter of the above decompositions. Therefore the resulting four-dimensional gauge group is $H = C_{E_8}(\mathrm{SU}(2) \times \mathrm{U}(1)) = E_6 \times \mathrm{U}(1)$. The $R = \mathrm{SU}(2) \times \mathrm{U}(1)$ content of the $\mathrm{Sp}(4)/(\mathrm{SU}(2) \times \mathrm{U}(1))_{\mathrm{non-max}}$ vector and spinor according to table 2 are $1_2 + 1_{-2} + 2_1 + 2_{-1}$ and $1_0 + 1_{-2} + 2_1$ respectively. Thus applying the CSDR rules (2.24), (2.25) and (2.26), (2.27) we find that the surviving fields in four dimensions can be organized in a $\mathcal{N} = 1$ vector supermultiplet V^α which transforms as 78 of E_6, a $\mathcal{N} = 1$ U(1) vector supermultiplet V and two chiral supermultiplets (B^i, C^i), transforming as $(27, 1)$ and $(27, -2)$ under $E_6 \times \mathrm{U}(1)$.

To determine the potential we have to go in the details and examine further the decomposition of the adjoint the specific S under R, i.e.

$$\begin{aligned} \mathrm{Sp}(4) &\supset (\mathrm{SU}(2) \times \mathrm{U}(1))_{\mathrm{non-max}} \\ 10 &= 3_0 + 1_0 + 1_2 + 1_{-2} + 2_1 + 2_{-1} \,. \end{aligned} \tag{3.22}$$

Then, according to the decomposition (3.22) the generators of Sp(4) can be grouped as follows,

$$Q_{\mathrm{Sp}(4)} = \{Q^\rho, Q, Q_+, Q^+, Q^a, Q_a\} \,, \tag{3.23}$$

where ρ takes values $1, 2, 3$ and a takes the values $1, 2$. The non-trivial commutation relations of the Sp(4) generators given in (3.23) are given in table 5. Furthermore the decomposition (3.23) suggests the following change in the notation of the scalar fields

$$\{\phi_I, I = 1, \ldots, 10\} \longrightarrow (\phi^\rho, \phi, \phi_+, \phi^+, \phi^a, \phi_a) \,, \tag{3.24}$$

which facilitates the solution of the constraints (2.33).

$\left[G^\alpha, G^\beta\right] = 2ig^{\alpha\beta\gamma}G^\gamma$	$\left[G^\rho, G^\sigma\right] = 2i\epsilon^{\rho\sigma\tau}G^\tau$
$\left[G, G^a\right] = \sqrt{3}G^a$	$\left[G, G^j\right] = -(2/\sqrt{3})G^j$
$\left[G, G^{aj}\right] = (1/\sqrt{3})G^{aj}$	$\left[G^\rho, G^a\right] = -(\tau^\rho)^a_b G^b$
$\left[G^\alpha, G^i\right] = -(G^\alpha)^i_j G^j$	$\left[G^\alpha, G^{ai}\right] = -(G^\alpha)^i_j G^{aj}$
$\left[G^a, G^j\right] = \sqrt{2}G^{aj}$	$\left[G_a, G^{bj}\right] = \sqrt{2}\delta^b_a G^j$
$\left[G^i, G^{aj}\right] = (-\sqrt{5/7})\epsilon^{ab}d^{ijk}G_{bk}$	$\left[G^{ai}, G^{bj}\right] = (\sqrt{5/7})\epsilon^{ab}d^{ijk}G_k$
$\left[G^i, G_{aj}\right] = \sqrt{2}\delta^i_j G_a$	$\left[G^a, G_b\right] = \sqrt{3}\delta^a_b - (\tau^\rho)^a_b G^\rho$
$\left[G^i, G_j\right] = (-2/\sqrt{3})\delta^i_j G + (G^\alpha)^i_j G^\alpha$	$\left[G^{ai}, G_{bj}\right] = (1/\sqrt{3})\delta^i_j\delta^a_b G + \delta^a_b (G^\alpha)^i_j G^\alpha - \delta^i_j (\tau^\rho)^a_b G^\rho$

Table 6: Non-trivial commutation relations of E_8 according to the decomposition given in eq. (3.27). The normalization is as follows $\mathrm{Tr}(G^\rho G_\sigma) = 2\delta^{\rho\sigma}$, $\mathrm{Tr}(G^\alpha G^\beta) = 12\delta^{\alpha\beta}$, $\mathrm{Tr}(G^a G_b) = 2\delta^a_b \mathrm{Tr}(GG) = 2$, $\mathrm{Tr}(G^i G_j) = 2\delta^i_j$, $\mathrm{Tr}(G^{ai}G_{bj}) = 2\delta^a_b\delta^i_j$.

On the other hand the potential of any gauge theory reduced over the coset space $\mathrm{Sp}(4)/(\mathrm{SU}(2)\times \mathrm{U}(1))_{\mathrm{non-max}}$ was found [2] to be in terms of the redefined fields in (3.24),

$$\begin{aligned} V(\phi) = {} & \frac{2\Lambda^2+6}{R_1^4} + \frac{4\Lambda^2}{R_2^4} + \frac{2}{R_2^4}\,\mathrm{Tr}(\phi_+\phi^+) + \frac{2}{R_1^4}\,\mathrm{Tr}(\phi_a\phi^a) - \frac{2\Lambda}{R_2^4}\,\mathrm{Tr}(Q[\phi_+,\phi^+]) - \\ & -\frac{\Lambda}{R_1^4}\,\mathrm{Tr}(Q[\phi_a,\phi^a]) - \frac{1}{R_1^4}(\tau_\rho)^a_b\,\mathrm{Tr}(Q_\rho[\phi_a,\phi^b]) \times \\ & \times\left[-\frac{\sqrt{2}}{R_1^2}\left(\frac{1}{R_2^2}+\frac{1}{2R_1^2}\right)\epsilon^{ab}\,\mathrm{Tr}(\phi_+[\phi_a,\phi_b]) + \mathrm{h.c.}\right] + \frac{1}{2}\left(\frac{1}{R_2^2}[\phi_+,\phi^+] + \frac{1}{R_1^2}[\phi_a,\phi^a]\right)^2 - \\ & -\frac{2}{R_1^2R_2^2}\,\mathrm{Tr}([\phi_+,\phi_a][\phi^+,\phi^a]) - \frac{1}{R_1^4}\,\mathrm{Tr}([\phi_a,\phi_b][\phi^a,\phi^b]), \end{aligned} \tag{3.25}$$

where, R_1 and R_2 are the coset space radii. In terms of the radii the real metric[1] of the coset space is

$$g_{ab} = \mathrm{diag}(R_1^2, R_1^2, R_2^2, R_2^2, R_1^2, R_1^2)\,. \tag{3.26}$$

To proceed we use the embedding (3.21) of $\mathrm{SU}(2)\times\mathrm{U}(1)$ in E_8 and divide its generators accordingly,

$$Q_{E_8} = \{G^\rho, G, G^\alpha, G^a, G_a, G^i, G_i, G^{ai}, G_{ai}\}\,, \tag{3.27}$$

where $\rho = 1,2,3$, $a = 1,2$, $\alpha = 1,\ldots,78$, $i = 1,\ldots,27$. The non-trivial commutation relations of the E_8 generators grouped in (3.27) are given in table 6.

Now the constraints (2.33) for the redefined fields in (3.24) become

$$[\phi,\phi_+] = 2\phi_+\,, \qquad [\phi,\phi_a] = \phi_a\,, \qquad [\phi_\rho,\phi_a] = (\tau_\rho)^b_a\phi_b\,. \tag{3.28}$$

Then we can write the solutions of the constraints (3.28) in terms of the genuine Higgs fields β^i, γ^i and the E_8 generators (3.27) corresponding to the embedding (3.21) as follows,

$$\begin{aligned} \phi^\rho &= G^\rho\,, \qquad \phi = \sqrt{3}G\,, \\ \phi_a &= R_1\frac{1}{\sqrt{2}}\beta^i G_i\,, \qquad \phi_+ = R_2\gamma^i G_i\,. \end{aligned} \tag{3.29}$$

[1] The coset space can be considered as a complex three dimensional space having coordinate indices $a,+$ with $a = 1,2$ and metric at $y = 0$ $g^{1\bar{1}} = g^{2\bar{2}} = 1/R_1^2$ and $g^{+\bar{+}} = 1/R_2^2$. The latter metric has been used to write the potential in the form given in eq. (3.25).

The potential (3.25) in terms of the physical scalar fields β^i and γ^i becomes

$$\begin{aligned}V(\beta,\gamma) &= \text{const} - \frac{6}{R_1^2}\beta^2 - \frac{4}{R_2^2}\gamma^2 + \left[4\sqrt{\frac{10}{7}}R_2\left(\frac{1}{R_2^2}+\frac{1}{2R_1^2}\right)d_{ijk}\beta^i\beta^j\gamma^k + \text{h.c.}\right] + \\ &\quad +6\Big(\beta^i(G^\alpha)_i^j\beta_j + \gamma^i(G^\alpha)_i^j\gamma_j\Big)^2 + \frac{1}{3}\Big(\beta^i(1\delta_i^j)\beta_j + \gamma^i(-2\delta_i^j)\gamma_j\Big)^2 + \\ &\quad +\frac{5}{7}\beta^i\beta^j d_{ijk}d^{klm}\beta_l\beta_m + 4\frac{5}{7}\beta^i\gamma^j d_{ijk}d^{klm}\beta_l\gamma_m\,. \end{aligned} \tag{3.30}$$

From the potential (3.30) we read the F-, D- and scalar soft terms as in the previous model. The F-terms can be derived from the superpotential

$$\mathcal{W}(B^i,C^j) = \sqrt{\frac{5}{7}}d_{ijk}B^iB^jC^k\,. \tag{3.31}$$

The D-term contributions are the sum

$$\frac{1}{2}D^\alpha D^\alpha + \frac{1}{2}DD\,, \tag{3.32}$$

where

$$D^\alpha = \sqrt{12}(\beta^i(G^\alpha)_i^j\beta_j + \gamma^i(G^\alpha)_i^j\gamma_j)$$

and

$$D = \sqrt{\frac{2}{3}}(\beta^i(1\delta_i^j)\beta_j + \gamma^i(-2\delta_i^j)\gamma_j)$$

corresponding to $E_6\times \mathrm{U}(1)$. The rest terms in the potential (3.30) are the soft breaking mass and trilinear terms and they form the scalar SSB part of the lagrangian,

$$\mathcal{L}_{\text{scalarSSB}} = -\frac{6}{R_1^2}\beta^2 - \frac{4}{R_2^2}\gamma^2 + \left[4\sqrt{\frac{10}{7}}R_2\left(\frac{1}{R_2^2}+\frac{1}{2R_1^2}\right)d_{ijk}\beta^i\beta^j\gamma^k + \text{h.c.}\right]. \tag{3.33}$$

The gaugino mass has been calculated in ref. [24] to be

$$M = (1+3\tau)\frac{R_2^2+2R_1^2}{8R_1^2R_2}\,. \tag{3.34}$$

We note that the chosen embedding of $R = \mathrm{SU}(2)\times\mathrm{U}(1)$ in E_8 satisfies the condition (2.30) which guarantees the renormalizability of the four dimensional theory, while the absence of any other term that does not belong to the supersymmetric $E_6\times\mathrm{U}(1)$ theory or to its SSB sector guarantees the improved ultraviolet behaviour of the theory as in the previous model. Finally note the contribution of the torsion in the gaugino mass (3.34).

3.2.3 Soft supersymmetry breaking by reduction over SU(3)/(U(1) × U(1))

In this model the only difference as compared to the previous ones is that the chosen coset space to reduce the same theory is the non-symmetric $B = \mathrm{SU}(3)/\,\mathrm{U}(1)\times\mathrm{U}(1)$. The decompositions to be used are

$$E_8 \supset \mathrm{SU}(2)\times\mathrm{U}(1)\times E_6 \supset \mathrm{U}(1)\times\mathrm{U}(1)\times E_6\,.$$

The 248 of E_8 is decomposed under $\mathrm{SU}(2)\times\mathrm{U}(1)$ according to (3.21) whereas the decomposition under $\mathrm{U}(1)\times\mathrm{U}(1)$ is the following:

$$\begin{aligned}248 = {} & 1_{(0,0)} + 1_{(0,0)} + 1_{(3,1/2)} + 1_{(-3,1/2)} + 1_{(0,-1)} + 1_{(0,1)} + 1_{(-3,-1/2)} + 1_{(3,-1/2)} + \\ & +78_{(0,0)} + 27_{(3,1/2)} + 27_{(-3,1/2)} + 27_{(0,-1)} + +\overline{27}_{(-3,-1/2)} + \overline{27}_{(3,-1/2)} + \overline{27}_{(0,1)}\,.\end{aligned} \tag{3.35}$$

In the present case R is chosen to be identified with the $\mathrm{U}(1)\times\mathrm{U}(1)$ of the latter decomposition. Therefore the resulting four-dimensional gauge group is

$$H = C_{E_8}(\mathrm{U}(1)\times\mathrm{U}(1)) = \mathrm{U}(1)\times\mathrm{U}(1)\times E_6\,.$$

The $R = \mathrm{U}(1)\times\mathrm{U}(1)$ content of $\mathrm{SU}(3)/\,\mathrm{U}(1)\times\mathrm{U}(1)$ vector and spinor are according to table 2,

$$\left(3,\frac{1}{2}\right) + \left(-3,\frac{1}{2}\right) + (0,-1) + \left(-3,-\frac{1}{2}\right) + \left(3,-\frac{1}{2}\right) + (0,1)$$

and

$$(0,0) + \left(3,\frac{1}{2}\right) + \left(-3,\frac{1}{2}\right) + (0,-1)$$

respectively. Thus applying the CSDR rules we find that the surviving fields in four dimensions are three $\mathcal{N}=1$ vector multiplets $V^\alpha, V_{(1)}, V_{(2)}$, (where α is an E_6, 78 index and the other two refer to the two $\mathrm{U}(1)'s$) containing the gauge fields of $\mathrm{U}(1)\times\mathrm{U}(1)\times E_6$. The matter content consists of three $\mathcal{N}=1$ chiral multiplets (A^i, B^i, C^i) with i an E_6, 27 index and three $\mathcal{N}=1$ chiral multiplets (A, B, C) which are E_6 singlets and carry $\mathrm{U}(1)\times\mathrm{U}(1)$ charges.

To determine the potential we examine further the decomposition of the adjoint of the specific $S = \mathrm{SU}(3)$ under $R = \mathrm{U}(1)\times\mathrm{U}(1)$, i.e.

$$\begin{aligned}&\mathrm{SU}(3) \supset \mathrm{U}(1)\times\mathrm{U}(1)\\ &\quad 8 = (0,0) + (0,0) + \left(3,\frac{1}{2}\right) + \left(-3,\frac{1}{2}\right) + (0,-1) + \left(-3,-\frac{1}{2}\right) + \left(3,-\frac{1}{2}\right) + (0,1)\,.\end{aligned} \tag{3.36}$$

Then according to the decomposition (3.36) the generators of SU(3) can be grouped as

$$Q_{\mathrm{SU}(3)} = \{Q_0, Q_0', Q_1, Q_2, Q_3, Q^1, Q^2, Q^3\}\,. \tag{3.37}$$

The non trivial commutator relations of SU(3) generators (3.37) are given in table 7. The decomposition (3.37) suggests the following change in the notation of the scalar fields,

$$(\phi_I, I=1,\ldots,8) \longrightarrow (\phi_0, \phi_0', \phi_1, \phi^1, \phi_2, \phi^2, \phi_3, \phi^3)\,. \tag{3.38}$$

The potential of any theory reduced over $\mathrm{SU}(3)/\,\mathrm{U}(1)\times\mathrm{U}(1))$ is given in terms of the redefined fields in (3.38) by

$$\begin{aligned}V(\phi) = {} & (3\Lambda^2+\Lambda'^2)\left(\frac{1}{R_1^4}+\frac{1}{R_2^4}\right) + \frac{4\Lambda'^2}{R_3^2} + \frac{2}{R_2^2R_3^2}\,\mathrm{Tr}(\phi_1\phi^1) + \frac{2}{R_1^2R_3^2}\,\mathrm{Tr}(\phi_2\phi^2) + \\ & + \frac{2}{R_1^2R_2^2}\,\mathrm{Tr}(\phi_3\phi^3) + \frac{\sqrt{3}\Lambda}{R_1^4}\,\mathrm{Tr}(Q_0[\phi_1,\phi^1]) - \frac{\sqrt{3}\Lambda}{R_2^4}\,\mathrm{Tr}(Q_0[\phi_2,\phi^2]) -\end{aligned}$$

$[Q_1, Q_0] = \sqrt{3}Q_1$	$[Q_1, Q'_0] = Q_1$	$[Q_2, Q_0] = -\sqrt{3}Q_2$
$[Q_2, Q'_0] = Q_2$	$[Q_3, Q_0] = 0$	$[Q_3, Q'_0] = -2Q_3$
$[Q_1, Q^1] = -\sqrt{3}Q_0 - Q'_0$	$[Q_2, Q^2] = \sqrt{3}Q_0 - Q'_0$	$[Q_3, Q^3] = 2Q'_0$
$[Q_1, Q_2] = \sqrt{2}Q^3$	$[Q_2, Q_3] = \sqrt{2}Q^1$	$[Q_3, Q_1] = \sqrt{2}Q^2$

Table 7: Non-trivial commutation relations of SU(3) according to the decomposition given in eq. (3.37). The normalization is $\mathrm{Tr}(Q_0Q_0) = \mathrm{Tr}(Q'_0Q'_0) = \mathrm{Tr}(Q_1Q^1) = \mathrm{Tr}(Q_2Q^2) = \mathrm{Tr}(Q_3Q^3) = 2$.

$[Q_1, Q_0] = \sqrt{30}Q_1$	$[Q_1, Q'_0] = \sqrt{10}Q_1$	$[Q_2, Q_0] = -\sqrt{30}Q_2$
$[Q_2, Q'_0] = \sqrt{10}Q_2$	$[Q_3, Q_0] = 0$	$[Q_3, Q'_0] = -2\sqrt{10}Q_3$
$[Q_1, Q^1] = -\sqrt{30}Q_0 - \sqrt{10}Q'_0$	$[Q_2, Q^2] = \sqrt{30}Q_0 - \sqrt{10}Q'_0$	$[Q_3, Q^3] = 2\sqrt{10}Q'_0$
$[Q_1, Q_2] = \sqrt{20}Q^3$	$[Q_2, Q_3] = \sqrt{20}Q^1$	$[Q_3, Q_1] = \sqrt{20}Q^2$
$[Q_{1i}, Q_0] = \sqrt{30}Q_{1i}$	$[Q_{1i}, Q'_0] = \sqrt{10}Q_{1i}$	$[Q_{2i}, Q_0] = -\sqrt{30}Q_{2i}$
$[Q_{2i}, Q'_0] = \sqrt{10}Q_{2i}$	$[Q_{3i}, Q_0] = 0$	$[Q_{3i}, Q'_0] = -2\sqrt{10}Q_{3i}$
$[Q_{1i}, Q_{2j}] = \sqrt{20}d_{ijk}Q^{3k}$	$[Q_{2i}, Q_{3j}] = \sqrt{20}d_{ijk}Q^{1k}$	$[Q_{3i}, Q_{1j}] = \sqrt{20}d_{ijk}Q^{2k}$
$[Q^\alpha, Q^\beta] = 2ig^{\alpha\beta\gamma}Q^\gamma$	$[Q^\alpha, Q_0] = 0$	$[Q^\alpha, Q'_0] = 0$
$[Q^\alpha, Q_{1i}] = -(G^\alpha)_i^jQ_{1j}$	$[Q^\alpha, Q_{2i}] = -(G^\alpha)_i^jQ_{2j}$	$[Q^\alpha, Q_{3i}] = -(G^\alpha)_i^jQ_{3j}$

Table 8: Non-trivial commutation relations of E_8 according to the decomposition given by eq. (3.41).

$$
\begin{aligned}
&-\frac{\sqrt{3}\Lambda}{R_3^4}\,\mathrm{Tr}(Q_0[\phi_3,\phi^3]) + \frac{\Lambda'}{R_1^4}\,\mathrm{Tr}(Q'_0[\phi_1,\phi^1]) + \frac{\Lambda'}{R_2^4}\,\mathrm{Tr}(Q'_0[\phi_2,\phi^2]) - \\
&-\frac{2\Lambda'}{R_3^4}\,\mathrm{Tr}(Q'_0[\phi_3,\phi^3]) + \\
&+\left[\frac{2\sqrt{2}}{R_1^2R_2^2}\,\mathrm{Tr}(\phi_3[\phi_1,\phi_2]) + \frac{2\sqrt{2}}{R_1^2R_3^2}\,\mathrm{Tr}(\phi_2[\phi_3,\phi_1]) + \frac{2\sqrt{2}}{R_2^2R_3^2}\,\mathrm{Tr}(\phi_1[\phi_2,\phi_3]) + \mathrm{h.c.}\right] + \\
&+\frac{1}{2}\,\mathrm{Tr}\left(\frac{1}{R_1^2}([\phi_1,\phi^1]) + \frac{1}{R_2^2}([\phi_2,\phi^2]) + \frac{1}{R_3^2}([\phi_3,\phi^3])\right)^2 - \\
&-\frac{1}{R_1^2R_2^2}\,\mathrm{Tr}([\phi_1,\phi_2][\phi^1,\phi^2]) - \frac{1}{R_1^2R_3^2}\,\mathrm{Tr}([\phi_1,\phi_3][\phi^1,\phi^3]) - \\
&-\frac{1}{R_2^2R_3^2}\,\mathrm{Tr}([\phi_2,\phi_3][\phi^2,\phi^3])\,,
\end{aligned}
\tag{3.39}
$$

where R_1, R_2, R_3 are the coset space radii.[2] In terms of the radii the real metric[3] of the coset is

$$
g_{ab} = \mathrm{diag}(R_1^2, R_1^2, R_2^2, R_2^2, R_3^2, R_3^2)\,. \tag{3.40}
$$

Next we examine the commutation relations of E_8 under the decomposition (3.35). Under this decomposition the generators of E_8 can be grouped as

$$
Q_{E_8} = \{Q_0, Q'_0, Q_1, Q_2, Q_3, Q^1, Q^2, Q^3, Q^\alpha, Q_{1i}, Q_{2i}, Q_{3i}, Q^{1i}, Q^{2i}, Q^{3i}\}\,, \tag{3.41}
$$

where $\alpha = 1, \ldots, 78$ and $i = 1, \ldots, 27$. The non-trivial commutation relations of the E_8

[2] To bring the potential into this form we have used (A.22) of ref. [2] and relations (7), (8) of ref. [33].

[3] The complex metric that was used is $g^{1\bar{1}} = 1/R_1^2$, $g^{2\bar{2}} = 1/R_2^2$, $g^{3\bar{3}} = 1/R_3^2$.

$$\boxed{\begin{array}{c}[Q_{1i},Q^{1j}]=-\frac{1}{6}(G^\alpha)_i^jQ^\alpha-\sqrt{30}\delta_i^jQ_0-\sqrt{10}\delta_i^jQ_0'\\ [Q_{2i},Q^{2j}]=-\frac{1}{6}(G^\alpha)_i^jQ^\alpha+\sqrt{30}\delta_i^jQ_0-\sqrt{10}\delta_i^jQ_0'\\ [Q_{3i},Q^{3j}]=-\frac{1}{6}(G^\alpha)_i^jQ^\alpha+2\sqrt{10}\delta_i^jQ_0'\end{array}}$$

Table 9: Further non-trivial commutation relations of E_8 according to the decomposition given in eq. (3.41). The normalization is $\mathrm{Tr}(Q_0Q_0)=\mathrm{Tr}(Q_0'Q_0')=\mathrm{Tr}(Q_1Q^1)=\mathrm{Tr}(Q_2Q^2)=\mathrm{Tr}(Q_3Q^3)=2$. $\mathrm{Tr}(Q_{1i}Q^{1j})=\mathrm{Tr}(Q_{2i}Q^{2j})=\mathrm{Tr}(Q_{3i}Q^{3j})=2\delta_i^j$. $\mathrm{Tr}(Q^\alpha Q^\beta)=12\delta^{\alpha\beta}$.

generators (3.41) are given in tables 8 and 9. Now the constraints (2.33) for the redefined fields in (3.38) are,

$$\begin{aligned} [\phi_1,\phi_0] &= \sqrt{3}\phi_1\,, & [\phi_1,\phi_0'] &= \phi_1\,,\\ [\phi_2,\phi_0] &= -\sqrt{3}\phi_2\,, & [\phi_2,\phi_0'] &= \phi_2\,,\\ [\phi_3,\phi_0] &= 0\,, & [\phi_3,\phi_0'] &= -2\phi_3\,. \end{aligned} \tag{3.42}$$

The solutions of the constraints (3.42) in terms of the genuine Higgs fields and of the E_8 generators (3.41) corresponding to the embedding (3.35) of $R=\mathrm{U}(1)\times\mathrm{U}(1)$ in the E_8 are, $\phi_0=\Lambda Q_0$ and $\phi_0'=\Lambda Q_0'$,with $\Lambda=\Lambda'=\frac{1}{\sqrt{10}}$, and

$$\begin{aligned} \phi_1 &= R_1\alpha^iQ_{1i}+R_1\alpha Q_1\,,\\ \phi_2 &= R_2\beta^iQ_{2i}+R_2\beta Q_2\,,\\ \phi_3 &= R_3\gamma^iQ_{3i}+R_3\gamma Q_3\,, \end{aligned} \tag{3.43}$$

where the unconstrained scalar fields transform under $\mathrm{U}(1)\times\mathrm{U}(1)\times E_6$ as

$$\begin{aligned} \alpha_i &\sim 27_{(3,1/2)}\,, & \alpha &\sim 1_{(3,1/2)}\,,\\ \beta_i &\sim 27_{(-3,1/2)}\,, & \beta &\sim 1_{(-3,1/2)}\,,\\ \gamma_i &\sim 27_{(0,-1)}\,, & \gamma &\sim 1_{(0,-1)}\,. \end{aligned} \tag{3.44}$$

The potential (3.39) becomes

$$\begin{aligned} V(\alpha,\beta,\gamma) =&\ \text{const.}+\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\alpha^i\alpha_i+\left(\frac{4R_1^2}{R_2^2R_3^2}-\frac{8}{R_1^2}\right)\overline{\alpha}\alpha+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\beta^i\beta_i+\\ &+\left(\frac{4R_2^2}{R_1^2R_3^2}-\frac{8}{R_2^2}\right)\overline{\beta}\beta+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\gamma^i\gamma_i+\left(\frac{4R_3^2}{R_1^2R_2^2}-\frac{8}{R_3^2}\right)\overline{\gamma}\gamma+\\ &+\Bigg[\sqrt{2}80\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)d_{ijk}\alpha^i\beta^j\gamma^k+\\ &\quad+\sqrt{2}80\left(\frac{R_1}{R_2R_3}+\frac{R_2}{R_1R_3}+\frac{R_3}{R_2R_1}\right)\alpha\beta\gamma+\text{h.c.}\Bigg]+\\ &+\frac{1}{6}\left(\alpha^i(G^\alpha)_i^j\alpha_j+\beta^i(G^\alpha)_i^j\beta_j+\gamma^i(G^\alpha)_i^j\gamma_j\right)^2+\\ &+\frac{10}{6}\left(\alpha^i(3\delta_i^j)\alpha_j+\overline{\alpha}(3)\alpha+\beta^i(-3\delta_i^j)\beta_j+\overline{\beta}(-3)\beta\right)^2+\\ &+\frac{40}{6}\left(\alpha^i\left(\frac{1}{2}\delta_i^j\right)\alpha_j+\overline{\alpha}\left(\frac{1}{2}\right)\alpha+\beta^i\left(\frac{1}{2}\delta_i^j\right)\beta_j+\overline{\beta}\left(\frac{1}{2}\right)\beta+\gamma^i\left(-1\delta_i^j\right)\gamma_j+\overline{\gamma}(-1)\gamma\right)^2+\\ &+40\alpha^i\beta^jd_{ijk}d^{klm}\alpha_l\beta_m+40\beta^i\gamma^jd_{ijk}d^{klm}\beta_l\gamma_m+40\alpha^i\gamma^jd_{ijk}d^{klm}\alpha_l\gamma_m+\\ &+40(\overline{\alpha}\overline{\beta})(\alpha\beta)+40(\overline{\beta}\overline{\gamma})(\beta\gamma)+40(\overline{\gamma}\overline{\alpha})(\gamma\alpha)\,. \end{aligned} \tag{3.45}$$

From the potential (3.45) we read the F-, D- and scalar soft terms. The F-terms are obtained from the superpotential

$$\mathcal{W}(A^i, B^j, C^k, A, B, C) = \sqrt{40} d_{ijk} A^i B^j C^k + \sqrt{40} ABC \,. \tag{3.46}$$

The D-terms have the structure

$$\frac{1}{2} D^\alpha D^\alpha + \frac{1}{2} D_1 D_1 + \frac{1}{2} D_2 D_2 \,, \tag{3.47}$$

where

$$\begin{aligned} D^\alpha &= \frac{1}{\sqrt{3}} \left(\alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j \right) , \\ D_1 &= \sqrt{\frac{10}{3}} \left(\alpha^i (3\delta_i^j) \alpha_j + \overline{\alpha}(3)\alpha + \beta^i (-3\delta_i^j) \beta_j + \overline{\beta}(-3)\beta \right) \end{aligned}$$

and

$$D_2 = \sqrt{\frac{40}{3}} \left(\alpha^i \left(\frac{1}{2}\delta_i^j\right) \alpha_j + \overline{\alpha}\left(\frac{1}{2}\right)\alpha + \beta^i \left(\frac{1}{2}\delta_i^j\right) \beta_j + \overline{\beta}\left(\frac{1}{2}\right)\beta + \gamma^i (-1\delta_i^j) \gamma_j + \overline{\gamma}(-1)\gamma \right)$$

which correspond to the $E_6 \times \mathrm{U}(1)_1 \times \mathrm{U}(1)_2$ structure of the gauge group. The rest terms are the trilinear and mass terms which break supersymmetry softly and they form the scalar SSB part of the lagrangian,

$$\begin{aligned} \mathcal{L}_{\text{scalarSSB}} = & \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \alpha^i \alpha_i + \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2} \right) \overline{\alpha}\alpha + \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \beta^i \beta_i + \\ & + \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2} \right) \overline{\beta}\beta + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \gamma^i \gamma_i + \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2} \right) \overline{\gamma}\gamma + \\ & + \left[\sqrt{280} \left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) d_{ijk} \alpha^i \beta^j \gamma^k + \right. \\ & \left. \quad + \sqrt{280} \left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_2 R_1} \right) \alpha\beta\gamma + \text{h.c.} \right] . \end{aligned} \tag{3.48}$$

Note that the potential (3.45) belongs to the case analyzed in subsection 2.3 where S has an image in G. Here $S = \mathrm{SU}(3)$ has an image in $G = E_8$ [32] so we conclude that the minimum of the potential is zero. Finally in order to determine the gaugino mass, we calculate the V operator using appendix A. We find that the gauginos acquire a geometrical mass

$$M = (1 + 3\tau) \frac{(R_1^2 + R_2^2 + R_3^2)}{8\sqrt{R_1^2 R_2^2 R_3^2}} \,. \tag{3.49}$$

Note again that the chosen embedding satisfies the condition (2.30) and the absence in the four-dimensional theory of any other term that does not belong to the supersymmetric $E_6 \times \mathrm{U}(1) \times \mathrm{U}(1)$ gauge theory or to its SSB sector. The gaugino mass (3.49), as in the two previous models, has a contribution from the torsion of the coset space.

A final remark concerning the gaugino masses in all three models reduced over six-dimensional non-symmetric coset spaces with torsion is that the adjustments required to

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obtain the *canonical connection* lead also to vanishing gaugino masses. Contrary to the gaugino mass term the soft scalar terms of the SSB do not receive contributions from the torsion. This is due to the fact that gauge fields, contrary to fermions, do not couple to torsion.

Concluding the present subsection, we would like to note that the fact that, starting with a $\mathcal{N} = 1$ supersymmetric theory in ten dimensions, the CSDR leads to the field content of an $\mathcal{N} = 1$ supersymmetric theory in the case that the six-dimensional coset spaces used are non-symmetric, can been seen by inspecting the table 2. More specifically, one notices in table 2 that when the coset spaces are non-symmetric the decompositions of the spinor 4 and antispinor $\overline{4}$ of SO(6) under R contain a singlet, i.e. have the form $1 + r$ and $1+\overline{r}$, respectively, where r is possibly reducible. The singlet under R provides the four-dimensional theory with fermions transforming according to the adjoint as was emphasized in subsection 2.3 and correspond to gauginos, which obtain geometrical and torsion mass contributions as we have seen in all three cases of the present subsection 3.2. Next turning the decomposition of the vector 6 of SO(6) under R in the non-symmetric cases, we recall that the vector can be constructed from the tensor product 4×4 and therefore has the form $r + \overline{r}$. Then the CSDR constraints tell us that the four-dimensional theory will contain the same representations of fermions and scalars since both come from the adjoint representation of the gauge group G and they have to satisfy the same matching conditions under R. Therefore the field content of the four-dimensional theory is, as expected, $\mathcal{N} = 1$ supersymmetric. To find out that furthermore the $\mathcal{N} = 1$ supersymmetry is softly broken, requires the lengthy and detailed analysis that was done above.

4. Conclusions

The CSDR was originally introduced as a scheme which, making use of higher dimensions, incorporates in a unified manner the gauge and the ad-hoc Higgs sector of the spontaneously broken gauge theories in four dimensions [1]. Next fermions were introduced in the scheme and the ad-hoc Yukawa interactions have also been included in the unified description [4, 6].

Of particular interest for the construction of fully realistic theories in the framework of CSDR are the following virtues that complemented the original suggestion: (i) The possibility to obtain chiral fermions in four dimensions resulting from vector-like representations of the higher dimensional gauge theory [6, 2]. This possibility can be realized due the presence of non-trivial background gauge configurations which are introduced by the CSDR constructions [34], (ii) The possibility to deform the metric of certain non-symmetric coset spaces and thereby obtain more than one scales [23, 2, 35], (iii) The possibility to use coset spaces, which are multiply connected. This can be achieved by exploiting the discrete symmetries of the S/R [36, 37, 2]. Then one might introduce topologically non-trivial gauge field [36] configurations with vanishing field strength and induce additional breaking of the gauge symmetry. It is the Hosotani mechanism [38] applied in the CSDR.

In the above list recently has been added the interesting possibility that the popular softly broken supersymmetric four-dimensional chiral gauge theories might have their origin

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in a higher dimensional supersymmetric theory with only a vector supermultiplet [7], which is dimensionally reduced over non-symmetric coset spaces.

In the present paper we have presented explicit and detailed examples of CSDR of a supersymmetric E_8 gauge theory over all possible six-dimensional coset spaces. Out of our study there are two cases that single out for further study as candidates to describe realistically the observed low energy world. Both are known GUTs containing three fermion families and scalars appropriate for the spontaneous electroweak breaking. One case is based on the reduction of the E_8 over the symmetric coset space $\mathrm{SU}(3) \times \mathrm{SU}(2)/\,\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$ and leads to an SO(10)-type non supersymmetric GUT in four dimensions. The other is based of the reduction of the same ten-dimensional gauge group over the non-symmetric coset space $\mathrm{SU}(3)/\,\mathrm{U}(1) \times \mathrm{U}(1)$ and leads to an E_6-type softly broken supersymmetric GUT in four dimensions. Both require some additional mechanism to break the four-dimensional GUT gauge group. Such a possibility is offered by the Hosotani mechanism, mentioned already, and in both cases there exist discrete symmetries acting freely on the corresponding coset spaces that can been used. We plan to return with a complete analysis of the possibilities to extract viable phenomenology from both models.

The current discussion on the higher dimensional theories with large extra dimensions provides a new framework to examine further the CSDR. An obvious advantage is a reexamination of CSDR over symmetric coset spaces. The fact that the four-dimensional scalar potential obtained from the reduction over symmetric coset spaces is tachyonic and appropriate for the electroweak symmetry breaking excludes the possibility of radii with size of the order of inverse of GUT or Planck scales, contrary to radii of inverse TeV scale. Similarly it is worth reexamining the cases that S can be embedded in the higher dimensional gauge group G and therefore the final gauge group after spontaneous symmetry breaking can be determined group theoretically. Again the spontaneous symmetry breaking is appropriate for the electroweak symmetry breaking, while there are not known examples that such a breaking is suitable for the GUT breaking. These latter cases provide also the advantage that the resulting four-dimensional theory has vanishing cosmological constant. Finally the classical treatment used in CSDR is justified in the case of large radii which are far away from the scales that the quantum effects of gravity are important.

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A. Calculation of the V operator

Here we give some details related to the calculation of the V operator in the case of $\mathrm{SU}(3)/\,\mathrm{U}(1) \times \mathrm{U}(1)$ and the gaugino mass (3.49).

To calculate the V operator in the case of $\mathrm{SU}(3)/\,\mathrm{U}(1) \times \mathrm{U}(1)$ we use the real metric of the coset, $g_{ab} = \mathrm{diag}(a, a, b, b, c, c)$ with $a = R_1^2, b = R_2^2, c = R_3^2$. Using the structure constants of SU(3), ${f_{12}}^3 = 2$, ${f_{45}}^8 = {f_{67}}^8 = \sqrt{3}$, ${f_{24}}^6 = {f_{14}}^7 = {f_{25}}^7 = -{f_{36}}^7 = -{f_{15}}^6 =$

$-f_{34}{}^5 = 1$, (where the indices 3 and 8 correspond to the $\mathrm{U}(1) \times \mathrm{U}(1)$ and the rest are the coset indices) we calculate the components of the D_{abc}:

$$\begin{aligned}
D_{523} &= D_{613} = D_{624} = D_{541} = -D_{514} = -D_{532} = -D_{631} = -D_{624} = \frac{1}{2}(c-a-b)\,, \\
D_{235} &= D_{136} = D_{624} = D_{154} = -D_{145} = -D_{253} = -D_{163} = -D_{264} = \frac{1}{2}(a-b-c)\,, \\
D_{352} &= D_{361} = D_{462} = D_{415} = -D_{451} = -D_{325} = -D_{316} = -D_{426} = \frac{1}{2}(b-c-a)\,,
\end{aligned}$$

From the D's we calculate the contorsion tensor

$$\Sigma_{abc} = 2\tau(D_{abc} + D_{bca} - D_{cba}),$$

and then the tensor

$$G_{abc} = D_{abc} + \frac{1}{2}\Sigma_{abc}$$

which is

$$\begin{aligned}
G_{523} &= G_{613} = G_{642} = G_{541} = -G_{514} = -G_{532} = -G_{631} = -G_{642} = \\
&= \frac{1}{2}[(1-\tau)c - (1+\tau)a - (1+\tau)b]\,, \\
G_{235} &= G_{136} = G_{246} = G_{154} = -G_{145} = -G_{253} = -G_{163} = -G_{264} = \\
&= \frac{1}{2}[-(1-\tau)a + (1+\tau)b + (1+\tau)c]\,, \\
G_{352} &= G_{361} = G_{462} = G_{415} = -G_{451} = -G_{325} = -G_{316} = -G_{426} = \\
&= \frac{1}{2}[-(1+\tau)a + (1-\tau)b - (1+\tau)c]\,.
\end{aligned}$$

In addition we need the gamma matrices. In ten dimensions we have $\{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$ with $\Gamma^\mu = \gamma^\mu \otimes I_8$ and $\{\Gamma^a, \Gamma^b\} = -2g^{ab}$, where

$$\Gamma^a = \frac{1}{\sqrt{r_a}}\gamma_5 \otimes \begin{bmatrix} 0 & \overline{\gamma}^a \\ \overline{\gamma}^a & 0 \end{bmatrix}$$

with $a = 1, 2, 3, 5, 6$ and

$$\Gamma^4 = \frac{1}{\sqrt{r_4}}\gamma_5 \otimes \begin{bmatrix} 0 & iI_4 \\ iI_4 & 0 \end{bmatrix}.$$

In the present case we have $r_1 = r_2 = a$, $r_3 = r_4 = b$ and $r_5 = r_6 = c$. The $\overline{\gamma}^a$ matrices are given by $\overline{\gamma}^1 = \sigma^1 \otimes \sigma^2$, $\overline{\gamma}^2 = \sigma^2 \otimes \sigma^2$, $\overline{\gamma}^3 = -I_2 \otimes \sigma^3$, $\overline{\gamma}^5 = \sigma^3 \otimes \sigma^2$, $\overline{\gamma}^6 = -I_2 \otimes \sigma^1$. Using these matrices we calculate $\Sigma^{ab} = \frac{1}{4}[\Gamma^a, \Gamma^b]$ and then $G_{abc}\Gamma^a\Sigma^{bc}$.

References

[1] P. Forgacs and N.S. Manton, *Space-time symmetries in gauge theories*, *Commun. Math. Phys.* **72** (1980) 15;
E. Witten, *Some exact multipseudoparticle solutions of classical Yang-Mills theory*, *Phys. Rev. Lett.* **38** (1977) 121.

[2] D. Kapetanakis and G. Zoupanos, *Coset space dimensional reduction of gauge theories*, *Phys. Rept.* **219** (1992) 1.

[3] Y. Kubyshin, J. M. Mourao, G. Rudolph and I. P. Volobujev, *Lecture notes in Physics*, **Vol. 349**, Springer Verlag, Heidelberg 1989.

[4] N.S. Manton, *Fermions and parity violation in dimensional reduction schemes*, *Nucl. Phys.* **B 193** (1981) 502.

[5] See e.g. M. B. Green, J. H. Schwarz and E. Witten, *Superstring theory*, Cambridge University Press, Cambridge 1987;
D. Luest and S. Theisen, *Lectures on string theory*, Lecture Notes in Physics, **Vol. 346**, Springer Verlag, Heidelberg 1989.

[6] G. Chapline and R. Slansky, *Dimensional reduction and flavor chirality*, *Nucl. Phys.* **B 209** (1982) 461.

[7] P. Manousselis and G. Zoupanos, *Supersymmetry breaking by dimensional reduction over coset spaces*, *Phys. Lett.* **B 504** (2001) 122 [hep-ph/0010141]; *Soft supersymmetry breaking due to dimensional reduction over non-symmetric coset spaces*, *Phys. Lett.* **B 518** (2001) 171 [hep-ph/0106033].

[8] S. Dimopoulos and H. Georgi, *Solution of the gauge hierarchy problem*, *Phys. Lett.* **B 117** (1982) 287;
N. Sakai, *Naturalness in supersymmetric "GUTs"*, *Z. Physik* **C 11** (1981) 153.

[9] D. Kapetanakis, M. Mondragón, and G. Zoupanos, *Finite unified models*, *Z. Physik* **C 60** (1993) 181 [hep-ph/9210218];
M. Mondragón, and G. Zoupanos, *Finite unified theories and the top quark mass*, *Nucl. Phys.* **37C** *(Proc. Suppl.)* (1995) 98.

[10] J. Kubo, M. Mondragón and G. Zoupanos, *Reduction of couplings and heavy top quark in the minimal SUSY GUT*, *Nucl. Phys.* **B 424** (1994) 291;
Perturbative unification of soft supersymmetry-breaking terms, *Phys. Lett.* **B 389** (1996) 523 [hep-ph/9609218];
T. Kobayashi, J. Kubo, M. Mondragon and G. Zoupanos, *Constraints on finite soft supersymmetry-breaking terms*, *Nucl. Phys.* **B 511** (1998) 45 [hep-ph/9707425];
For an extended discussion and a complete list of references see J. Kubo, M. Mondragón, and G. Zoupanos, *Unification beyond GUTS: gauge Yukawa unification*, *Acta Phys. Polon.* **B27** (1997) 3911 [hep-ph/9703289].

[11] See e.g. T. Kobayashi et.al., *Exact finite and gauge-Yukawa unified theories and their predictions*, *Acta Phys. Polon.* **B30** (1999) 2013; Proc. of HEP99, Tempere 1999, p.804.

[12] T.R. Taylor and G. Veneziano, *Strings and $D = 4$*, *Phys. Lett.* **B 212** (1988) 147.

[13] K.R. Dienes, E. Dudas and T. Gherghetta, *Grand unification at intermediate mass scales through extra dimensions*, *Nucl. Phys.* **B 537** (1999) 47 [hep-ph/9806292].

[14] T. Kobayashi, J. Kubo, M. Mondragon and G. Zoupanos, *Running of soft parameters in extra space-time dimensions*, *Nucl. Phys.* **B 550** (1999) 99 [hep-ph/9812221];
J. Kubo, H. Terao and G. Zoupanos, *Kaluza-klein thresholds and regularization (in)dependence*, *Nucl. Phys.* **B 574** (2000) 495 [hep-ph/9910277]; *Running couplings in extra dimensions*, hep-ph/0010069.

[15] L. Castellani, *On G/H geometry and its use in M-theory compactifications*, *Ann. Phys. (NY)* **287** (2001) 1 [hep-th/9912277].

[16] D. Luest, *Compactification of ten-dimensional superstring theories over Ricci flat coset spaces*, *Nucl. Phys.* **B 276** (1986) 220;
L. Castellani and D. Lüst, *Superstring compactification on homogeneous coset spaces with torsion*, *Nucl. Phys.* **B 296** (1988) 143.

[17] A.M. Gavrilik, *Coset-space string compactification leading to 14 subcritical dimensions*, hep-th/9911120.

[18] F. Muller-Hoissen and R. Stuckl, *Coset spaces and ten-dimensional unified theories*, *Class. and Quant. Grav.* **5** (1988) 27;
N.A. Batakis, K. Farakos, D. Kapetanakis, G. Koutsoumbas and G. Zoupanos, *Compactification over coset spaces with torsion and vanishing cosmological constant*, *Phys. Lett.* **B 220** (1989) 513.

[19] C. Wetterich, *Dimensional reduction of Weyl, Majorana and Majorana-Weyl spinors*, *Nucl. Phys.* **B 222** (1983) 20;
L. Palla, *On dimensional reduction of gauge theories: symmetric fields and harmonic expansion*, *Z. Physik* **C 24** (1983) 195;
K. Pilch and A.N. Schellekens, *Formulae for the eigenvalues of the laplacian on tensor harmonics on symmetric coset spaces*, *J. Math. Phys.* **25** (1984) 3455;
P. Forgacs, Z. Horvath and L. Palla, *Spontaneous compactification to nonsymmetric spaces*, *Z. Physik* **C 30** (1986) 261;
K.J. Barnes, P. Forgacs, M. Surridge and G. Zoupanos, *On fermion masses in a dimensional reduction scheme*, *Z. Physik* **C 33** (1987) 427.

[20] E. Witten, *Some properties of* O(32) *superstrings*, *Phys. Lett.* **B 149** (1984) 351.

[21] K. Pilch and A. N. Schellekens, *Fermion spectra from superstrings*, *Nucl. Phys.* **B 259** (1985) 637;
D. Luest, *Compactification of ten-dimensional superstring theories over Ricci flat coset spaces*, *Nucl. Phys.* **B 276** (1985) 220;
D. Kapetanakis and G. Zoupanos, *A unified theory in higher dimensions*, *Phys. Lett.* **B 249** (1990) 66.

[22] G. Chapline and N.S. Manton, *The geometrical significance of certain Higgs potentials: an approach to grand unification*, *Nucl. Phys.* **B 184** (1981) 391;
K. Farakos, G. Koutsoumbas, M. Surridge and G. Zoupanos, *Dimensional reduction and the Higgs potential*, *Nucl. Phys.* **B 291** (1987) 128;
G. Zoupanos, *Dilaton and dynamical fermion mass generation*, *Phys. Lett.* **B 190** (1987) 127;
K. Farakos, G. Koutsoumbas, M. Surridge and G. Zoupanos, *Geometrical hierarchy and spontaneous symmetry breaking*, *Phys. Lett.* **B 191** (1987) 135;
Y.A. Kubyshin, J.M. Mourao and I.P. Volobuev, *Scalar fields in the dimensional reduction scheme for symmetric spaces*, *Int. J. Mod. Phys.* **A 4** (1989) 151;
F.A. Bais, K.J. Barnes, P. Forgacs and G. Zoupanos, *Dimensional reduction of gauge theories yielding unified models spontaneously broken to* $\mathrm{SU}(3) \times \mathrm{U}(1)$, *Nucl. Phys.* **B 263** (1986) 557.

[23] K. Farakos, G. Koutsoumbas, M. Surridge and G. Zoupanos, *Dimensional reduction and the Higgs potential*, *Nucl. Phys.* **B 291** (1987) 128; *Geometrical hierarchy and spontaneous symmetry breaking*, *Phys. Lett.* **B 191** (1987) 135.

[24] D. Kapetanakis and G. Zoupanos, *Fermion masses from dimensional reduction*, *Phys. Lett.* **B 249** (1990) 73; *Constructing unified models based on E_8 in ten-dimensions*, *Z. Physik* **C 56** (1992) 91.

[25] See e.g. I. Antoniadis and K. Benakli, *Large dimensions and string physics in future colliders*, *Int. J. Mod. Phys.* **A 15** (2000) 4237 [hep-ph/0007226] and references therein;
Nima Arkani-Hamed, S. Dimopoulos and G. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263; *Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity*, *Phys. Rev.* **D 59** (1999) 086004 [hep-ph/9807344];
G.R. Dvali, S. Randjbar-Daemi and R. Tabbash, *The origin of spontaneous symmetry breaking in theories with large extra dimensions*, *Phys. Rev.* **D 65** (2002) 064021 [hep-ph/0102307];
Y.A. Kubyshin, *Models with extra dimensions and their phenomenology*, hep-ph/0111027.

[26] E. Fahri and L. Susskind, *Technicolor*, *Phys. Rept.* **C74** (1981) 277;
C.T. Hill, *Topcolor*, Proc. of Nagoya Int. Workshop (1996), p. 54, [hep-ph/9702320] and references therein; *Topcolor: top quark condensation in a gauge extension of the standard model*, *Phys. Lett.* **B 266** (1991) 419; *Topcolor assisted technicolor*, *Phys. Lett.* **B 345** (1995) 483 [hep-ph/9411426].

[27] G. Zoupanos, *Fermion mass generation and electroweak symmetry breaking from color forces*, *Phys. Lett.* **B 129** (1983) 315;
P. Forgacs and G. Zoupanos, *Dimensional reduction and dynamical symmetry breaking*, *Phys. Lett.* **B 148** (1984) 99;
D. Luest et al., *Phenomenology of high color fermions*, *Nucl. Phys.* **B 268** (1986) 49.

[28] G. Triantaphyllou, *New physics with mirror particles*, *J. Phys.* **G 26** (2000) 99 [hep-ph/9811250];
G. Triantaphyllou and G. Zoupanos, *Strongly interacting fermions from a higher-dimensional unified gauge theory*, *Phys. Lett.* **B 489** (2000) 420 [hep-ph/0006262];
G. Triantaphyllou, *Mass generation and the dynamical role of the katoptron group*, *Mod. Phys. Lett.* **A 16** (2001) 53 [hep-ph/0010147].

[29] M.B. Green and J.H. Schwarz, *Anomaly cancellation in supersymmetric $D = 10$ gauge theory and superstring theory*, *Phys. Lett.* **B 149** (1984) 117;
L. Alvarez-Gaumé and E. Witten, *Gravitational anomalies*, *Nucl. Phys.* **B 234** (1984) 269.

[30] P. Hořava and E. Witten, *Heterotic and type I string dynamics from eleven dimensions*, *Nucl. Phys.* **B 460** (1996) 506 [hep-th/9510209];
E. Witten, *Strong coupling expansion of Calabi-Yau compactification*, *Nucl. Phys.* **B 471** (1996) 135 [hep-th/9602070];
for a review see C. Muñoz, *Effective supergravity from heterotic M-theory and its phenomenological implications*, hep-th/9906152.

[31] T.W. Kephart and M.T. Vaughn, *Tensor methods for the exceptional group E_6*, *Ann. Phys. (NY)* **145** (1983) 162.

[32] D. Lüst and G. Zoupanos, *Dimensional reduction of ten-dimensional E_8 gauge theory over a compact coset space S/R*, *Phys. Lett.* **B 165** (1985) 309.

[33] E. Witten, *Dimensional reduction of superstring models*, *Phys. Lett.* **B 155** (1985) 151.

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[34] P. Forgacs, Z. Horvath and L. Palla, *Stable compactifying Einstein Yang-Mills systems*, *Phys. Lett.* **B 147** (1984) 311;
P. Forgacs and G. Zoupanos, *Dimensional reduction and dynamical symmetry breaking*, *Phys. Lett.* **B 148** (1984) 99;
R. Coquereaux and A. Jadczyk, *Symmetries of Einstein Yang-Mills fields and dimensional reduction*, *Commun. Math. Phys.* **98** (1985) 79;
A.N. Schellekens, *Stability of higher dimensional Einstein Yang-Mills theories on symmetric spaces*, *Nucl. Phys.* **B 248** (1984) 706;
S. Randjbar-Daemi, A. Salam and J. Strathdee, *Stability of instanton induced compactification in eight- dimensions*, *Nucl. Phys.* **B 242** (1984) 447; *Instability of higher dimensional Yang-Mills systems*, *Phys. Lett.* **B 124** (1983) 345.

[35] B.E. Hanlon and G.C. Joshi, *A three generation unified model from coset space dimensional reduction*, *Phys. Lett.* **B 298** (1993) 312.

[36] G. Zoupanos, *Wilson flux breaking and coset space dimensional reduction*, *Phys. Lett.* **B 201** (1988) 301.

[37] N.G. Kozimirov, V.A. Kuzmin and I.I. Tkachev, *Dimensional reduction on not simply connected homogeneous spaces*, *Sov. J. Nucl. Phys.* **49** (1989) 164;
D. Kapetanakis and G. Zoupanos, *Discrete symmetries and coset space dimensional reduction*, *Phys. Lett.* **B 232** (1989) 104.

[38] Y. Hosotani, *Dynamical mass generation by compact extra dimensions*, *Phys. Lett.* **B 126** (1983) 309; *Dynamical gauge symmetry breaking as the Casimir effect*, *Phys. Lett.* **B 129** (1983) 193;
E. Witten, *Symmetry breaking patterns in superstring models*, *Nucl. Phys.* **B 258** (1985) 75;
J.D. Breit, B.A. Ovrut and G.C. Segre, *E_6 symmetry breaking in the superstring theory*, *Phys. Lett.* **B 158** (1985) 33;
B. Greene, K. Kirlkin and P.J. Miron, *Superstring models with* SU(5) *and* SO(10) *unifying group*, *Nucl. Phys.* **B 274** (1986) 574; *A three generation superstring model. 1. compactification and discrete symmetries*, *Nucl. Phys.* **B 278** (1986) 667.